EXERCISES IN EUCLID.

I. 1 to 15.

1. On a given straight line describe an isosceles triangle having each of the sides equal to a given straight line.

2. In the figure of I. 2 if the diameter of the smaller circle is the radius of the larger, shew where the given point and the vertex of the constructed triangle will be situated.

3. If two straight lines bisect each other at right angles, any point in either of them is equidistant from the extremities of the other.

4. If the angles ABC and ACB at the base of an isosceles triangle be bisected by the straight lines BD, CD, shew that DBC will be an isosceles triangle.

5. BAC is a triangle having the angle B double of the angle A. If BD bisects the angle B and meets AC at D, shew that BD is equal to AD.

6. In the figure of I. 5 if FC and BG meet at H shew that FH and GH are equal.

7. In the figure of I. 5 if FC and BG meet at H, shew that AH bisects the angle BAC.

8. The sides AB, AD of a quadrilateral $ABCD$ are equal, and the diagonal AC bisects the angle BAD: shew that the sides CB and CD are equal, and that the diagonal AC bisects the angle BCD.

9. ACB, ADB are two triangles on the same side of AB, such that AC is equal to BD, and AD is equal to BC, and AD and BC intersect at O: shew that the triangle AOB is isosceles.

10. The opposite angles of a rhombus are equal.

11. A diagonal of a rhombus bisects each of the angles through which it passes.

12. If two isosceles triangles are on the same base the straight line joining their vertices, or that straight line produced, will bisect the base at right angles.

13. Find a point in a given straight line such that its distances from two given points may be equal.

14. Through two given points on opposite sides of a given straight line draw two straight lines which shall meet in that given straight line, and include an angle bisected by that given straight line.

15. A given angle *BAC* is bisected; if *CA* is produced to *G* and the angle *BAG* bisected, the two bisecting lines are at right angles.

16. If four straight lines meet at a point so that the opposite angles are equal, these straight lines are two and two in the same straight line.

I. 16 to 26.

17. *ABC* is a triangle and the angle *A* is bisected by a straight line which meets *BC* at *D*; shew that *BA* is greater than *BD*, and *CA* greater than *CD*.

18. In the figure of I. 17 shew that *ABC* and *ACB* are together less than two right angles, by joining *A* to any point in *BC*.

19. *ABCD* is a quadrilateral of which *AD* is the longest side and *BC* the shortest; shew that the angle *ABC* is greater than the angle *ADC*, and the angle *BCD* greater than the angle *BAD*.

20. If a straight line be drawn through *A* one of the angular points of a square, cutting one of the opposite sides, and meeting the other produced at *F*, shew that *AF* is greater than the diagonal of the square.

21. The perpendicular is the shortest straight line that can be drawn from a given point to a given straight line; and of others, that which is nearer to the perpendicular is less than the more remote; and two, and only two, equal straight lines can be drawn from the given point to the given straight line, one on each side of the perpendicular.

22. The sum of the distances of any point from the three angles of a triangle is greater than half the sum of the sides of the triangle.

23. The four sides of any quadrilateral are together greater than the two diagonals together.

24. The two sides of a triangle are together greater than twice the straight line drawn from the vertex to the middle point of the base.

25. If one angle of a triangle is equal to the sum of the other two, the triangle can be divided into two isosceles triangles.

26. If the angle C of a triangle is equal to the sum of the angles A and B, the side AB is equal to twice the straight line joining C to the middle point of AB.

27. Construct a triangle, having given the base, one of the angles at the base, and the sum of the sides.

28. The perpendiculars let fall on two sides of a triangle from any point in the straight line bisecting the angle between them are equal to each other.

29. In a given straight line find a point such that the perpendiculars drawn from it to two given straight lines shall be equal.

30. Through a given point draw a straight line such that the perpendiculars on it from two given points may be on opposite sides of it and equal to each other.

31. A straight line bisects the angle A of a triangle ABC; from B a perpendicular is drawn to this bisecting straight line, meeting it at D, and BD is produced to meet AC or AC produced at E: shew that BD is equal to DE.

32. AB, AC are any two straight lines meeting at A: through any point P draw a straight line meeting them at E and F, such that AE may be equal to AF.

33. Two right-angled triangles have their hypotenuses equal, and a side of one equal to a side of the other: shew that they are equal in all respects.

I. 27 to 31.

34. Any straight line parallel to the base of an isosceles triangle makes equal angles with the sides.

35. If two straight lines A and B are respectively parallel to two others C and D, shew that the inclination of A to B is equal to that of C to D.

36. A straight line is drawn terminated by two parallel straight lines; through its middle point any straight line is

drawn and terminated by the parallel straight lines. Shew that the second straight line is bisected at the middle point of the first.

37. If through any point equidistant from two parallel straight lines, two straight lines be drawn cutting the parallel straight lines, they will intercept equal portions of these parallel straight lines.

38. If the straight line bisecting the exterior angle of a triangle be parallel to the base, shew that the triangle is isosceles.

39. Find a point B in a given straight line CD, such that if AB be drawn to B from a given point A, the angle ABC will be equal to a given angle.

40. If a straight line be drawn bisecting one of the angles of a triangle to meet the opposite side, the straight lines drawn from the point of section parallel to the other sides, and terminated by these sides, will be equal.

41. The side BC of a triangle ABC is produced to a point D; the angle ACB is bisected by the straight line CE which meets AB at E. A straight line is drawn through E parallel to BC, meeting AC at F, and the straight line bisecting the exterior angle ACD at G. Shew that EF is equal to FG.

42. AB is the hypotenuse of a right-angled triangle ABC: find a point D in AB such that DB may be equal to the perpendicular from D on AC.

43. ABC is an isosceles triangle: find points D, E in the equal sides AB, AC such that BD, DE, EC may all be equal.

44. A straight line drawn at right angles to BC the base of an isosceles triangle ABC cuts the side AB at D and CA produced at E: shew that AED is an isosceles triangle.

I. 32.

45. From the extremities of the base of an isosceles triangle straight lines are drawn perpendicular to the sides; shew that the angles made by them with the base are each equal to half the vertical angle.

46. On the sides of any triangle ABC equilateral triangles BCD, CAE, ABF are described, all external: shew that the straight lines AD, BE, CF are all equal.

47. What is the magnitude of an angle of a regular octagon?

48. Through two given points draw two straight lines forming with a straight line given in position an equilateral triangle.

49. If the straight lines bisecting the angles at the base of an isosceles triangle be produced to meet, they will contain an angle equal to an exterior angle of the triangle.

50. *A* is the vertex of an isosceles triangle *ABC*, and *BA* is produced to *D*, so that *AD* is equal to *BA*; and *DC* is drawn: shew that *BCD* is a right angle.

51. *ABC* is a triangle, and the exterior angles at *B* and *C* are bisected by the straight lines *BD*, *CD* respectively, meeting at *D*: shew that the angle *BDC* together with half the angle *BAC* make up a right angle.

52. Shew that any angle of a triangle is obtuse, right, or acute, according as it is greater than, equal to, or less than the other two angles of the triangle taken together.

53. Construct an isosceles triangle having the vertical angle four times each of the angles at the base.

54. In the triangle *ABC* the side *BC* is bisected at *E* and *AB* at *G*; *AE* is produced to *F* so that *EF* is equal to *AE*, and *CG* is produced to *H* so that *GH* is equal to *CG*: shew that *FB* and *HB* are in one straight line.

55. Construct an isosceles triangle which shall have one-third of each angle at the base equal to half the vertical angle.

56. *AB*, *AC* are two straight lines given in position: it is required to find in them two points *P* and *Q*, such that, *PQ* being joined, *AP* and *PQ* may together be equal to a given straight line, and may contain an angle equal to a given angle.

57. Straight lines are drawn through the extremities of the base of an isosceles triangle, making angles with it on the side remote from the vertex, each equal to one-third of one of the equal angles of the triangle and meeting the sides produced: shew that three of the triangles thus formed are isosceles.

58. *AEB*, *CED* are two straight lines intersecting at *E*; straight lines *AC*, *DB* are drawn forming two triangles *ACE*, *BED*; the angles *ACE*, *DBE* are bisected by the straight lines *CF*, *BF*, meeting at *F*. Shew that the angle *CFB* is equal to half the sum of the angles *EAC*, *EDB*.

59. The straight line joining the middle point of the hypotenuse of a right-angled triangle to the right angle is equal to half the hypotenuse.

60. From the angle A of a triangle ABC a perpendicular is drawn to the opposite side, meeting it, produced if necessary, at D; from the angle B a perpendicular is drawn to the opposite side, meeting it, produced if necessary, at E: shew that the straight lines which join D and E to the middle point of AB are equal.

61. From the angles at the base of a triangle perpendiculars are drawn to the opposite sides, produced if necessary: shew that the straight line joining the points of intersection will be bisected by a perpendicular drawn to it from the middle point of the base.

62. In the figure of I. 1, if C and H be the points of intersection of the circles, and AB be produced to meet one of the circles at K, shew that CHK is an equilateral triangle.

63. The straight lines bisecting the angles at the base of an isosceles triangle meet the sides at D and E: shew that DE is parallel to the base.

64. AB, AC are two given straight lines, and P is a given point in the former: it is required to draw through P a straight line to meet AC at Q, so that the angle APQ may be three times the angle AQP.

65. Construct a right-angled triangle, having given the hypotenuse and the sum of the sides.

66. Construct a right-angled triangle, having given the hypotenuse and the difference of the sides.

67. Construct a right-angled triangle, having given the hypotenuse and the perpendicular from the right angle on it.

68. Construct a right-angled triangle, having given the perimeter and an angle.

69. Trisect a right angle.

70. Trisect a given finite straight line.

71. From a given point it is required to draw to two parallel straight lines, two equal straight lines at right angles to each other.

72. Describe a triangle of given perimeter, having its angles equal to those of a given triangle.

I. 33, 34.

73. If a quadrilateral have two of its opposite sides parallel, and the two others equal but not parallel, any two of its opposite angles are together equal to two right angles.

74. If a straight line which joins the extremities of two equal straight lines, not parallel, make the angles on the same side of it equal to each other, the straight line which joins the other extremities will be parallel to the first.

75. No two straight lines drawn from the extremities of the base of a triangle to the opposite sides can possibly bisect each other.

76. If the opposite sides of a quadrilateral are equal it is a parallelogram.

77. If the opposite angles of a quadrilateral are equal it is a parallelogram.

78. The diagonals of a parallelogram bisect each other.

79. If the diagonals of a quadrilateral bisect each other it is a parallelogram.

80. If the straight line joining two opposite angles of a parallelogram bisect the angles the four sides of the parallelogram are equal.

81. Draw a straight line through a given point such that the part of it intercepted between two given parallel straight lines may be of given length.

82. Straight lines bisecting two adjacent angles of a parallelogram intersect at right angles.

83. Straight lines bisecting two opposite angles of a parallelogram are either parallel or coincident.

84. If the diagonals of a parallelogram are equal all its angles are equal.

85. Find a point such that the perpendiculars let fall from it on two given straight lines shall be respectively equal to two given straight lines. How many such points are there?

86. It is required to draw a straight line which shall be equal to one straight line and parallel to another, and be terminated by two given straight lines.

87. On the sides AB, BC, and CD of a parallelogram $ABCD$ three equilateral triangles are described, that on BC towards the same parts as the parallelogram, and those on AB, CD towards the opposite parts: shew that the

distances of the vertices of the triangles on *AB*, *CD* from that on *BC* are respectively equal to the two diagonals of the parallelogram.

88. If the angle between two adjacent sides of a parallelogram be increased, while their lengths do not alter, the diagonal through their point of intersection will diminish.

89. *A*, *B*, *C* are three points in a straight line, such that *AB* is equal to *BC*: shew that the sum of the perpendiculars from *A* and *C* on any straight line which does not pass between *A* and *C* is double the perpendicular from *B* on the same straight line.

90. If straight lines be drawn from the angles of any parallelogram perpendicular to any straight line which is outside the parallelogram, the sum of those from one pair of opposite angles is equal to the sum of those from the other pair of opposite angles.

91. If a six-sided plane rectilineal figure have its opposite sides equal and parallel, the three straight lines joining the opposite angles will meet at a point.

92. *AB*, *AC* are two given straight lines; through a given point *E* between them it is required to draw a straight line *GEH* such that the intercepted portion *GH* shall be bisected at the point *E*.

93. Inscribe a rhombus within a given rhombus, so that one of the angular points of the inscribed figure may bisect a side of the other.

94. *ABCD* is a parallelogram, and *E*, *F*, the middle points of *AD* and *BC* respectively; shew that *BE* and *DF* will trisect the diagonal *AC*.

I. 35 to 45.

95. *ABCD* is a quadrilateral having *BC* parallel to *AD*; shew that its area is the same as that of the parallelogram which can be formed by drawing through the middle point of *DC* a straight line parallel to *AB*.

96. *ABCD* is a quadrilateral having *BC* parallel to *AD*, *E* is the middle point of *DC*; shew that the triangle *AEB* is half the quadrilateral.

97. Shew that any straight line passing through the middle point of the diameter of a parallelogram and terminated by two opposite sides, bisects the parallelogram.

98. Bisect a parallelogram by a straight line drawn through a given point within it.

99. Construct a rhombus equal to a given parallelogram.

100. If two triangles have two sides of the one equal to two sides of the other, each to each, and the sum of the two angles contained by these sides equal to two right angles, the triangles are equal in area.

101. A straight line is drawn bisecting a parallelogram *ABCD* and meeting *AD* at *E* and *BC* at *F*: shew that the triangles *EBF* and *CED* are equal.

102. Shew that the four triangles into which a parallelogram is divided by its diagonals are equal in area.

103. Two straight lines *AB* and *CD* intersect at *E*, and the triangle *AEC* is equal to the triangle *BED*: shew that *BC* is parallel to *AD*.

104. *ABCD* is a parallelogram; from any point *P* in the diagonal *BD* the straight lines *PA*, *PC* are drawn. Shew that the triangles *PAB* and *PCB* are equal.

105. If a triangle is described having two of its sides equal to the diagonals of any quadrilateral, and the included angle equal to either of the angles between these diagonals, then the area of the triangle is equal to the area of the quadrilateral.

106. The straight line which joins the middle points of two sides of any triangle is parallel to the base.

107. Straight lines joining the middle points of adjacent sides of a quadrilateral form a parallelogram.

108. *D*, *E* are the middle points of the sides *AB*, *AC* of a triangle, and *CD*, *BE* intersect at *F*: shew that the triangle *BFC* is equal to the quadrilateral *ADFE*.

109. The straight line which bisects two sides of any triangle is half the base.

110. In the base *AC* of a triangle take any point *D*; bisect *AD*, *DC*, *AB*, *BC* at the points *E*, *F*, *G*, *H* respectively: shew that *EG* is equal and parallel to *FH*.

111. Given the middle points of the sides of a triangle, construct the triangle.

112. If the middle points of any two sides of a triangle be joined, the triangle so cut off is one quarter of the whole.

113. The sides *AB*, *AC* of a given triangle *ABC* are bisected at the points *E*, *F*; a perpendicular is drawn from *A* to the opposite side, meeting it at *D*. Shew that the

angle *FDE* is equal to the angle *BAC*. Shew also that *AFDE* is half the triangle *ABC*.

114. Two triangles of equal area stand on the same base and on opposite sides: shew that the straight line joining their vertices is bisected by the base or the base produced.

115. Three parallelograms which are equal in all respects are placed with their equal bases in the same straight line and contiguous; the extremities of the base of the first are joined with the extremities of the side opposite to the base of the third, towards the same parts: shew that the portion of the new parallelogram cut off by the second is one half the area of any one of them.

116. *ABCD* is a parallelogram; from *D* draw any straight line *DFG* meeting *BC* at *F* and *AB* produced at *G*; draw *AF* and *CG*: shew that the triangles *ABF*, *CFG* are equal.

117. *ABC* is a given triangle: construct a triangle of equal area, having for its base a given straight line *AD*, coinciding in position with *AB*.

118. *ABC* is a given triangle: construct a triangle of equal area, having its vertex at a given point in *BC* and its base in the same straight line as *AB*.

119. *ABCD* is a given quadrilateral: construct another quadrilateral of equal area having *AB* for one side, and for another a straight line drawn through a given point in *CD* parallel to *AB*.

120. *ABCD* is a quadrilateral: construct a triangle whose base shall be in the same straight line as *AB*, vertex at a given point *P* in *CD*, and area equal to that of the given quadrilateral.

121. *ABC* is a given triangle: construct a triangle of equal area, having its base in the same straight line as *AB*, and its vertex in a given straight line parallel to *AB*.

122. Bisect a given triangle by a straight line drawn through a given point in a side.

123. Bisect a given quadrilateral by a straight line drawn through a given angular point.

124. If through the point *O* within a parallelogram *ABCD* two straight lines are drawn parallel to the sides, and the parallelograms *OB* and *OD* are equal, the point *O* is in the diagonal *AC*.

I. 46 to 48.

125. On the sides *AC*, *BC* of a triangle *ABC*, squares *ACDE*, *BCFH* are described: shew that the straight lines *AF* and *BD* are equal.

126. The square on the side subtending an acute angle of a triangle is less than the squares on the sides containing the acute angle.

127. The square on the side subtending an obtuse angle of a triangle is greater than the squares on the sides containing the obtuse angle.

128. If the square on one side of a triangle be less than the squares on the other two sides, the angle contained by these sides is an acute angle; if greater, an obtuse angle.

129. A straight line is drawn parallel to the hypotenuse of a right-angled triangle, and each of the acute angles is joined with the points where this straight line intersects the sides respectively opposite to them: shew that the squares on the joining straight lines are together equal to the square on the hypotenuse and the square on the straight line drawn parallel to it.

130. If any point *P* be joined to *A*, *B*, *C*, *D*, the angular points of a rectangle, the squares on *PA* and *PC* are together equal to the squares on *PB* and *PD*.

131. In a right-angled triangle if the square on one of the sides containing the right angle be three times the square on the other, and from the right angle two straight lines be drawn, one to bisect the opposite side, and the other perpendicular to that side, these straight lines divide the right angle into three equal parts.

132. If *ABC* be a triangle whose angle *A* is a right angle, and *BE*, *CF* be drawn bisecting the opposite sides respectively, shew that four times the sum of the squares on *BE* and *CF* is equal to five times the square on *BC*.

133. On the hypotenuse *BC*, and the sides *CA*, *AB* of a right-angled triangle *ABC*, squares *BDEC*, *AF*, and *AG* are described: shew that the squares on *DG* and *EF* are together equal to five times the square on *BC*.

II. 1 to 11.

134. A straight line is divided into two parts; shew that if twice the rectangle of the parts is equal to the sum of the squares described on the parts, the straight line is bisected.

135. Divide a given straight line into two parts such that the rectangle contained by them shall be the greatest possible.

136. Construct a rectangle equal to the difference of two given squares.

137. Divide a given straight line into two parts such that the sum of the squares on the two parts may be the least possible.

138. Shew that the square on the sum of two straight lines together with the square on their difference is double the squares on the two straight lines.

139. Divide a given straight line into two parts such that the sum of their squares shall be equal to a given square.

140. Divide a given straight line into two parts such that the square on one of them may be double the square on the other.

141. In the figure of II. 11 if *CH* be produced to meet *BF* at *L*, shew that *CL* is at right angles to *BF*.

142. In the figure of II. 11 if *BE* and *CH* meet at *O*, shew that *AO* is at right angles to *CH*.

143. Shew that in a straight line divided as in II. 11 the rectangle contained by the sum and difference of the parts is equal to the rectangle contained by the parts.

II. 12 to 14.

144. The square on the base of an isosceles triangle is equal to twice the rectangle contained by either side and by the straight line intercepted between the perpendicular let fall on it from the opposite angle and the extremity of the base.

145. In any triangle the sum of the squares on the sides is equal to twice the square on half the base together with twice the square on the straight line drawn from the vertex to the middle point of the base.

146. *ABC* is a triangle having the sides *AB* and *AC* equal; if *AB* is produced beyond the base to *D* so that *BD* is equal to *AB*, shew that the square on *CD* is equal to the square on *AB*, together with twice the square on *BC*.

147. The sum of the squares on the sides of a parallelogram is equal to the sum of the squares on the diagonals.

148. The base of a triangle is given and is bisected by the centre of a given circle: if the vertex be at any point of the circumference, shew that the sum of the squares on the two sides of the triangle is invariable.

149. In any quadrilateral the squares on the diagonals are together equal to twice the sum of the squares on the straight lines joining the middle points of opposite sides.

150. If a circle be described round the point of intersection of the diameters of a parallelogram as a centre, shew that the sum of the squares on the straight lines drawn from any point in its circumference to the four angular points of the parallelogram is constant.

151. The squares on the sides of a quadrilateral are together greater than the squares on its diagonals by four times the square on the straight line joining the middle points of its diagonals.

152. In *AB* the diameter of a circle take two points *C* and *D* equally distant from the centre, and from any point *E* in the circumference draw *EC*, *ED*: shew that the squares on *EC* and *ED* are together equal to the squares on *AC* and *AD*.

153. In *BC* the base of a triangle take *D* such that the squares on *AB* and *BD* are together equal to the squares on *AC* and *CD*, then the middle point of *AD* will be equally distant from *B* and *C*.

154. The square on any straight line drawn from the vertex of an isosceles triangle to the base is less than the square on a side of the triangle by the rectangle contained by the segments of the base.

155. A square *BDEC* is described on the hypotenuse *BC* of a right-angled triangle *ABC*: shew that the squares on *DA* and *AC* are together equal to the squares on *EA* and *AB*.

156. *ABC* is a triangle in which *C* is a right angle, and *DE* is drawn from a point *D* in *AC* perpendicular to

AB: shew that the rectangle *AB*, *AE* is equal to the rectangle *AC*, *AD*.

157. If a straight line be drawn through one of the angles of an equilateral triangle to meet the opposite side produced, so that the rectangle contained by the whole straight line thus produced and the part of it produced is equal to the square on the side of the triangle, shew that the square on the straight line so drawn will be double the square on a side of the triangle.

158. In a triangle whose vertical angle is a right angle a straight line is drawn from the vertex perpendicular to the base: shew that the square on this perpendicular is equal to the rectangle contained by the segments of the base.

159. In a triangle whose vertical angle is a right angle a straight line is drawn from the vertex perpendicular to the base: shew that the square on either of the sides adjacent to the right angle is equal to the rectangle contained by the base and the segment of it adjacent to that side.

160. In a triangle *ABC* the angles *B* and *C* are acute: if *E* and *F* be the points where perpendiculars from the opposite angles meet the sides *AC*, *AB*, shew that the square on *BC* is equal to the rectangle *AB*, *BF*, together with the rectangle *AC*, *CE*.

161. Divide a given straight line into two parts so that the rectangle contained by them may be equal to the square described on a given straight line which is less than half the straight line to be divided.

III. 1 to 15.

162. Describe a circle with a given centre cutting a given circle at the extremities of a diameter.

163. Shew that the straight lines drawn at right angles to the sides of a quadrilateral inscribed in a circle from their middle points intersect at a fixed point.

164. If two circles cut each other, any two parallel straight lines drawn through the points of section to cut the circles are equal.

165. Two circles whose centres are *A* and *B* intersect at *C*; through *C* two chords *DCE* and *FCG* are drawn equally inclined to *AB* and terminated by the circles: shew that *DE* and *FG* are equal.

166. Through either of the points of intersection of two given circles draw the greatest possible straight line terminated both ways by the two circumferences.

167. If from any point in the diameter of a circle straight lines are drawn to the extremities of a parallel chord, the squares on these straight lines are together equal to the squares on the segments into which the diameter is divided.

168. *A* and *B* are two fixed points without a circle *PQR*; it is required to find a point *P* in the circumference, so that the sum of the squares described on *AP* and *BP* may be the least possible.

169. If in any two given circles which touch one another, there be drawn two parallel diameters, an extremity of each diameter, and the point of contact, shall lie in the same straight line.

170. A circle is described on the radius of another circle as diameter, and two chords of the larger circle are drawn, one through the centre of the less at right angles to the common diameter, and the other at right angles to the first through the point where it cuts the less circle. Shew that these two chords have the segments of the one equal to the segments of the other, each to each.

171. Through a given point within a circle draw the shortest chord.

172. *O* is the centre of a circle, *P* is any point in its circumference, *PN* a perpendicular on a fixed diameter: shew that the straight line which bisects the angle *OPN* always passes through one or the other of two fixed points.

173. Three circles touch one another externally at the points *A*, *B*, *C*; from *A*, the straight lines *AB*, *AC* are produced to cut the circle *BC* at *D* and *E*: shew that *DE* is a diameter of *BC*, and is parallel to the straight line joining the centres of the other circles.

174. Circles are described on the sides of a quadrilateral as diameters: shew that the common chord of any adjacent two is parallel to the common chord of the other two.

175. Describe a circle which shall touch a given circle, have its centre in a given straight line, and pass through a given point in the given straight line.

III. 16 to 19.

176. Shew that two tangents can be drawn to a circle from a given external point, and that they are of equal length.

177. Draw parallel to a given straight line a straight line to touch a given circle.

178. Draw perpendicular to a given straight line a straight line to touch a given circle.

179. In the diameter of a circle produced, determine a point so that the tangent drawn from it to the circumference shall be of given length.

180. Two circles have the same centre: shew that all chords of the outer circle which touch the inner circle are equal.

181. Through a given point draw a straight line so that the part intercepted by the circumference of a given circle shall be equal to a given straight line not greater than the diameter.

182. Two tangents are drawn to a circle at the opposite extremities of a diameter, and cut off from a third tangent a portion AB: if C be the centre of the circle shew that ACB is a right angle.

183. Describe a circle that shall have a given radius and touch a given circle and a given straight line.

184. A circle is drawn to touch a given circle and a given straight line. Shew that the points of contact are always in the same straight line with a fixed point in the circumference of the given circle.

185. Draw a straight line to touch each of two given circles.

186. Draw a straight line to touch one given circle so that the part of it contained by another given circle shall be equal to a given straight line not greater than the diameter of the latter circle.

187. Draw a straight line cutting two given circles so that the chords intercepted within the circles shall have given lengths.

188. A quadrilateral is described so that its sides touch a circle: shew that two of its sides are together equal to the other two sides.

189. Shew that no parallelogram can be described about a circle except a rhombus.

190. *ABD*, *ACE* are two straight lines touching a circle at *B* and *C*, and if *DE* be joined *DE* is equal to *BD* and *CE* together: shew that *DE* touches the circle.

191. If a quadrilateral be described about a circle the angles subtended at the centre of the circle by any two opposite sides of the figure are together equal to two right angles.

192. Two radii of a circle at right angles to each other when produced are cut by a straight line which touches the circle: shew that the tangents drawn from the points of section are parallel to each other.

193. A straight line is drawn touching two circles: shew that the chords are parallel which join the points of contact and the points where the straight line through the centres meets the circumferences.

194. If two circles can be described so that each touches the other and three of the sides of a quadrilateral figure, then the difference between the sums of the opposite sides is double the common tangent drawn across the quadrilateral.

195. *AB* is the diameter and *C* the centre of a semicircle: shew that *O* the centre of any circle inscribed in the semicircle is equidistant from *C* and from the tangent to the semicircle parallel to *AB*.

196. If from any point without a circle straight lines be drawn touching it, the angle contained by the tangents is double the angle contained by the straight line joining the points of contact and the diameter drawn through one of them.

197. A quadrilateral is bounded by the diameter of a circle, the tangents at its extremities, and a third tangent: shew that its area is equal to half that of the rectangle contained by the diameter and the side opposite to it.

198. If a quadrilateral, having two of its sides parallel, be described about a circle, a straight line drawn through the centre of the circle, parallel to either of the two parallel sides, and terminated by the other two sides, shall be equal to a fourth part of the perimeter of the figure.

199. A series of circles touch a fixed straight line at a fixed point: shew that the tangents at the points where they cut a parallel fixed straight line all touch a fixed circle.

200. Of all straight lines which can be drawn from two given points to meet in the convex circumference of a

given circle, the sum of the two is least which make equal angles with the tangent at the point of concourse.

201. *C* is the centre of a given circle, *CA* a radius, *B* a point on a radius at right angles to *CA*; join *AB* and produce it to meet the circle again at *D*, and let the tangent at *D* meet *CB* produced at *E*: shew that *BDE* is an isosceles triangle.

202. Let the diameter *BA* of a circle be produced to *P*, so that *AP* equals the radius; through *A* draw the tangent *AED*, and from *P* draw *PEC* touching the circle at *C* and meeting the former tangent at *E*; join *BC* and produce it to meet *AED* at *D*: then will the triangle *DEC* be equilateral.

III. 20 to 22.

203. Two tangents *AB*, *AC* are drawn to a circle; *D* is any point on the circumference outside of the triangle *ABC*: shew that the sum of the angles *ABD* and *ACD* is constant.

204. *P*, *Q* are any points in the circumferences of two segments described on the same straight line *AB*, and on the same side of it; the angles *PAQ*, *PBQ* are bisected by the straight lines *AR*, *BR* meeting at *R*: shew that the angle *ARB* is constant.

205. Two segments of a circle are on the same base *AB*, and *P* is any point in the circumference of one of the segments; the straight lines *APD*, *BPC* are drawn meeting the circumference of the other segment at *D* and *C*; *AC* and *BD* are drawn intersecting at *Q*. Shew that the angle *AQB* is constant.

206. *APB* is a fixed chord passing through *P* a point of intersection of two circles *AQP*, *PBR*; and *QPR* is any other chord of the circles passing through *P*: shew that *AQ* and *RB* when produced meet at a constant angle.

207. *AOB* is a triangle; *C* and *D* are points in *BO* and *AO* respectively, such that the angle *ODC* is equal to the angle *OBA*: shew that a circle may be described round the quadrilateral *ABCD*.

208. $ABCD$ is a quadrilateral inscribed in a circle, and the sides AB, CD when produced meet at O: shew that the triangles AOC, BOD are equiangular.

209. Shew that no parallelogram except a rectangle can be inscribed in a circle.

210. A triangle is inscribed in a circle: shew that the sum of the angles in the three segments exterior to the triangle is equal to four right angles.

211. A quadrilateral is inscribed in a circle: shew that the sum of the angles in the four segments of the circle exterior to the quadrilateral is equal to six right angles.

212. Divide a circle into two parts so that the angle contained in one segment shall be equal to twice the angle contained in the other.

213. Divide a circle into two parts so that the angle contained in one segment shall be equal to five times the angle contained in the other.

214. If the angle contained by any side of a quadrilateral and the adjacent side produced, be equal to the opposite angle of the quadrilateral, shew that any side of the quadrilateral will subtend equal angles at the opposite angles of the quadrilateral.

215. If any two consecutive sides of a hexagon inscribed in a circle be respectively parallel to their opposite sides, the remaining sides are parallel to each other.

216. A, B, C, D are four points taken in order on the circumference of a circle; the straight lines AB, CD produced intersect at P, and AD, BC at Q: shew that the straight lines which respectively bisect the angles APC, AQC are perpendicular to each other.

217. If a quadrilateral be inscribed in a circle, and a straight line be drawn making equal angles with one pair of opposite sides, it will make equal angles with the other pair.

218. A quadrilateral can have one circle inscribed in it and another circumscribed about it: shew that the straight lines joining the opposite points of contact of the inscribed circle are perpendicular to each other.

III. 23 to 30.

219. The straight lines joining the extremities of the chords of two equal arcs of a circle, towards the same parts are parallel to each other.

220. The straight lines in a circle which join the extremities of two parallel chords are equal to each other.

221. *AB* is a common chord of two circles; through *C* any point of one circumference straight lines *CAD*, *CBE* are drawn terminated by the other circumference: shew that the arc *DE* is invariable.

222. Through a point *C* in the circumference of a circle two straight lines *ACB*, *DCE* are drawn cutting the circle at *B* and *E*: shew that the straight line which bisects the angles *ACE*, *DCB* meets the circle at a point equidistant from *B* and *E*.

223. The straight lines bisecting any angle of a quadrilateral inscribed in a circle and the opposite exterior angle, meet in the circumference of the circle.

224. *AB* is a diameter of a circle, and *D* is a given point on the circumference: draw a chord *DE* on one side of *AB* so that one arc between the chord and diameter may be three times the other.

225. From *A* and *B* two of the angular points of a triangle *ABC*, straight lines are drawn so as to meet the opposite sides at *P* and *Q* in given equal angles: shew that the straight line joining *P* and *Q* will be of the same length in all triangles on the same base *AB*, and having vertical angles equal to *C*.

226. If two equal circles cut each other, and if through one of the points of intersection a straight line be drawn terminated by the circles, the straight lines joining its extremities with the other point of intersection are equal.

227. *OA*, *OB*, *OC* are three chords of a circle; the angle *AOB* is equal to the angle *BOC*, and *OA* is nearer to the centre than *OB*. From *B* a perpendicular is drawn on *OA*, meeting it at *P*, and a perpendicular on *OC* produced, meeting it at *Q*: shew that *AP* is equal to *CQ*.

228. *AB* is a given finite straight line; through *A* two indefinite straight lines are drawn equally inclined to *AB*; any circle passing through *A* and *B* meets these straight lines at *L* and *M*. Shew that if *AB* be between *AL* and *AM* the sum of *AL* and *AM* is constant; if *AB* be not between *AL* and *AM* the difference of *AL* and *AM* is constant.

229. *AOB* and *COD* are diameters of a circle at right angles to each other; *E* is a point in the arc *AC*, and *EFG* is a chord meeting *COD* at *F*, and drawn in such a

direction that EF is equal to the radius. Shew that the arc BG is equal to three times the arc AE.

230. The straight lines which bisect the vertical angles of all triangles on the same base and on the same side of it, and having equal vertical angles, all intersect at the same point.

231. If two circles touch each other internally, any chord of the greater circle which touches the less shall be divided at the point of its contact into segments which subtend equal angles at the point of contact of the two circles.

III. 31.

232. Right-angled triangles are described on the same hypotenuse: shew that the angular points opposite the hypotenuse all lie on a circle described on the hypotenuse as diameter.

233. The circles described on the equal sides of an isosceles triangle as diameters, will intersect at the middle point of the base.

234. The greatest rectangle which can be inscribed in a circle is a square.

235. The hypotenuse AB of a right-angled triangle ABC is bisected at D, and EDF is drawn at right angles to AB, and DE and DF are cut off each equal to DA; CE and CF are joined: shew that the last two straight lines will bisect the angle C and its supplement respectively.

236. On the side AB of any triangle ABC as diameter a circle is described; EF is a diameter parallel to BC: shew that the straight lines EB and FB bisect the interior and exterior angles at B.

237. If AD, CE be drawn perpendicular to the sides BC, AB of a triangle ABC, and DE be joined, shew that the angles ADE and ACE are equal to each other.

238. If two circles ABC, ABD intersect at A and B, and AC, AD be two diameters, shew that the straight line CD will pass through B.

239. If O be the centre of a circle and OA a radius and a circle be described on OA as diameter, the circum-

ference of this circle will bisect any chord drawn through it from A to meet the exterior circle.

240. Describe a circle touching a given straight line at a given point, such that the tangents drawn to it from two given points in the straight line may be parallel.

241. Describe a circle with a given radius touching a given straight line, such that the tangents drawn to it from two given points in the straight line may be parallel.

242. If from the angles at the base of any triangle perpendiculars are drawn to the opposite sides, produced if necessary, the straight line joining the points of intersection will be bisected by a perpendicular drawn to it from the centre of the base.

243. AD is a diameter of a circle; B and C are points on the circumference on the same side of AD; a perpendicular from D on BC produced through C, meets it at E: shew that the square on AD is greater than the sum of the squares on AB, BC, CD, by twice the rectangle BC, CE.

244. AB is the diameter of a semicircle, P is a point on the circumference, PM is perpendicular to AB; on AM, BM as diameters two semicircles are described, and AP, BP meet these latter circumferences at Q, R: shew that QR will be a common tangent to them.

245. AB, AC are two straight lines, B and C are given points in the same; BD is drawn perpendicular to AC, and DE perpendicular to AB; in like manner CF is drawn perpendicular to AB, and FG to AC. Shew that EG is parallel to BC.

246. Two circles intersect at the points A and B, from which are drawn chords to a point C in one of the circumferences, and these chords, produced if necessary, cut the other circumference at D and E: shew that the straight line DE cuts at right angles that diameter of the circle ABC which passes through C.

247. If squares be described on the sides and hypotenuse of a right-angled triangle, the straight line joining the intersection of the diagonals of the latter square with the right angle is perpendicular to the straight line joining the intersections of the diagonals of the two former.

248. C is the centre of a given circle, CA a straight line less than the radius; find the point of the circumference at which CA subtends the greatest angle.

249. AB is the diameter of a semicircle, D and E are any two points in its circumference. Shew that if the chords joining A and B with D and E each way intersect at F and G, then FG produced is at right angles to AB.

250. Two equal circles touch one another externally, and through the point of contact chords are drawn, one to each circle, at right angles to each other: shew that the straight line joining the other extremities of these chords is equal and parallel to the straight line joining the centres of the circles.

251. A circle is described on the shorter diagonal of a rhombus as a diameter, and cuts the sides; and the points of intersection are joined crosswise with the extremities of that diagonal: shew that the parallelogram thus formed is a rhombus with angles equal to those of the first.

252. If two chords of a circle meet at a right angle within or without a circle, the squares on their segments are together equal to the squares on the diameter.

III. 32 to 34.

253. B is a point in the circumference of a circle, whose centre is C; PA, a tangent at any point P, meets CB produced at A, and PD is drawn perpendicular to CB: shew that the straight line PB bisects the angle APD.

254. If two circles touch each other, any straight line drawn through the point of contact will cut off similar segments.

255. AB is any chord, and AD is a tangent to a circle at A. DPQ is any straight line parallel to AB, meeting the circumference at P and Q. Shew that the triangle PAD is equiangular to the triangle QAB.

256. Two circles $ABDH$, ABG, intersect each other at the points A, B; from B a straight line BD is drawn in the one to touch the other; and from A any chord whatever is drawn cutting the circles at G and H: shew that BG is parallel to DH.

257. Two circles intersect at A and B. At A the tangents AC, AD are drawn to each circle and terminated

by the circumference of the other. If *CB*, *BD* be joined, shew that *AB* or *AB* produced, if necessary, bisects the angle *CBD*.

258. Two circles intersect at *A* and *B*, and through *P* any point in the circumference of one of them the chords *PA* and *PB* are drawn to cut the other circle at *C* and *D*: shew that *CD* is parallel to the tangent at *P*.

259. If from any point in the circumference of a circle a chord and tangent be drawn, the perpendiculars dropped on them from the middle point of the subtended arc are equal to one another.

260. *AB* is any chord of a circle, *P* any point on the circumference of the circle; *PM* is a perpendicular on *AB* and is produced to meet the circle at *Q*; and *AN* is drawn perpendicular to the tangent at *P*: shew that the triangle *NAM* is equiangular to the triangle *PAQ*.

261. Two diameters *AOB*, *COD* of a circle are at right angles to each other; *P* is a point in the circumference; the tangent at *P* meets *COD* produced at *Q*, and *AP*, *BP* meet the same line at *R*, *S* respectively: shew that *RQ* is equal to *SQ*

262. Construct a triangle, having given the base, the vertical angle, and the point in the base on which the perpendicular falls.

263. Construct a triangle, having given the base, the vertical angle, and the altitude.

264. Construct a triangle, having given the base, the vertical angle, and the length of the straight line drawn from the vertex to the middle point of the base.

265. Having given the base and the vertical angle of a triangle, shew that the triangle will be greatest when it is isosceles.

266. From a given point *A* without a circle whose centre is *O* draw a straight line cutting the circle at the points *B* and *C*, so that the area *BOC* may be the greatest possible.

267. Two straight lines containing a constant angle always pass through two fixed points, their position being otherwise unrestricted: shew that the straight line bisecting the angle always passes through one or other of two fixed points.

268. Given one angle of a triangle, the side opposite

it, and the sum of the other two sides, construct the triangle.

III. 35 to 37.

269. If two circles cut one another, the tangents drawn to the two circles from any point in the common chord produced are equal.

270. Two circles intersect at A and B: shew that AB produced bisects their common tangent.

271. If AD, CE are drawn perpendicular to the sides BC, AB of a triangle ABC, shew that the rectangle contained by BC and BD is equal to the rectangle contained by BA and BE.

272. If through any point in the common chord of two circles which intersect one another, there be drawn any two other chords, one in each circle, their four extremities shall all lie in the circumference of a circle.

273. From a given point as centre describe a circle cutting a given straight line in two points, so that the rectangle contained by their distances from a fixed point in the straight line may be equal to a given square.

274. Two circles $ABCD$, $EBCF$, having the common tangents AE and DF, cut one another at B and C, and the chord BC is produced to cut the tangents at G and H: shew that the square on GH exceeds the square on AE or DF by the square on BC.

275. A series of circles intersect each other, and are such that the tangents to them from a fixed point are equal: shew that the straight lines joining the two points of intersection of each pair will pass through this point.

276. ABC is a right-angled triangle; from any point D in the hypotenuse BC a straight line is drawn at right angles to BC, meeting CA at E and BA produced at F: shew that the square on DE is equal to the difference of the rectangles BD, DC and AE, EC; and that the square on DF is equal to the sum of the rectangles BD, DC and AF, FB.

277. It is required to find a point in the straight line which touches a circle at the end of a given diameter, such that when a straight line is drawn from this point to the other extremity of the diameter, the rectangle contained

by the part of it without the circle and the part within the circle may be equal to a given square not greater than that on the diameter.

IV. 1 to 4.

278. In IV. 3 shew that the straight lines drawn through *A* and *B* to touch the circle will meet.

279. In IV. 4 shew that the straight lines which bisect the angles *B* and *C* will meet.

280. In IV. 4 shew that the straight line *DA* will bisect the angle at *A*.

281. If the circle inscribed in a triangle *ABC* touch the sides *AB*, *AC* at the points *D*, *E*, and a straight line be drawn from *A* to the centre of the circle meeting the circumference at *G*, shew that the point *G* is the centre of the circle inscribed in the triangle *ADE*.

282. Shew that the straight lines joining the centres of the circles touching one side of a triangle and the others produced, pass through the angular points of the triangle.

283. A circle touches the side *BC* of a triangle *ABC* and the other two sides produced: shew that the distance between the points of contact of the side *BC* with this circle and with the inscribed circle, is equal to the difference between the sides *AB* and *AC*.

284. A circle is inscribed in a triangle *ABC*, and a triangle is cut off at each angle by a tangent to the circle. Shew that the sides of the three triangles so cut off are together equal to the sides of *ABC*.

285. *D* is the centre of the circle inscribed in a triangle *BAC*, and *AD* is produced to meet the straight line drawn through *B* at right angles to *BD* at *O*: shew that *O* is the centre of the circle which touches the side *BC* and the sides *AB*, *AC* produced.

286. Three circles are described, each of which touches one side of a triangle *ABC*, and the other two sides produced. If *D* be the point of contact of the side *BC*, *E* that of *AC*, and *F* that of *AB*, shew that *AE* is equal to *BD*, *BF* to *CE*, and *CD* to *AF*.

287. Describe a circle which shall touch a given circle and two given straight lines which themselves touch the given circle.

288. If the three points be joined in which the circle inscribed in a triangle meets the sides, shew that the resulting triangle is acute angled.

289. Two opposite sides of a quadrilateral are together equal to the other two, and each of the angles is less than two right angles. Shew that a circle can be inscribed in the quadrilateral.

290. Two circles *HPL*, *KPM*, that touch each other externally, have the common tangents *HK*, *LM*; *HL* and *KM* being joined, shew that a circle may be inscribed in the quadrilateral *HKML*.

291. Straight lines are drawn from the angles of a triangle to the centres of the opposite escribed circles: shew that these straight lines intersect at the centre of the inscribed circle.

292. Two sides of a triangle whose perimeter is constant are given in position: shew that the third side always touches a certain circle.

293. Given the base, the vertical angle, and the radius of the inscribed circle of a triangle, construct it.

IV. 5 to 9.

294. In IV. 5 shew that the perpendicular from *F* on *BC* will bisect *BC*.

295. If *DE* be drawn parallel to the base *BC* of a triangle *ABC*, shew that the circles described about the triangles *ABC* and *ADE* have a common tangent.

296. If the inscribed and circumscribed circles of a triangle be concentric, shew that the triangle must be equilateral.

297. Shew that if the straight line joining the centres of the inscribed and circumscribed circles of a triangle passes through one of its angular points, the triangle is isosceles.

298. The common chord of two circles is produced to any point *P*; *PA* touches one of the circles at *A*, *PBC* is any chord of the other. Shew that the circle which passes through *A*, *B*, and *C* touches the circle to which *PA* is a tangent.

299. A quadrilateral *ABCD* is inscribed in a circle, and *AD*, *BC* are produced to meet at *E*: shew that the circle described about the triangle *ECD* will have the tangent at *E* parallel to *AB*.

300. Describe a circle which shall touch a given straight line, and pass through two given points.

301. Describe a circle which shall pass through two given points and cut off from a given straight line a chord of given length.

302. Describe a circle which shall have its centre in a given straight line, and cut off from two given straight lines chords of equal given length.

303. Two triangles have equal bases and equal vertical angles: shew that the radius of the circumscribing circle of one triangle is equal to that of the other.

304. Describe a circle which shall pass through two given points, so that the tangent drawn to it from another given point may be of a given length.

305. C is the centre of a circle; CA, CB are two radii at right angles; from B any chord BP is drawn cutting CA at N: a circle being described round ANP, shew that it will be touched by BA.

306. AB and CD are parallel straight lines, and tho straight lines which join their extremities intersect at E: shew that the circles described round the triangles ABE, CDE touch one another.

307. Find the centre of a circle cutting off three equal chords from the sides of a triangle.

308. If O be the centre of the circle inscribed in the triangle ABC, and AO be produced to meet the circumscribed circle at F, show that FB, FO, and FC are all equal.

309. The opposite sides of a quadrilateral inscribed in a circle are produced to meet at P and Q, and about the triangles so formed without the quadrilateral, circles are described meeting again at R: shew that P, R, Q are in one straight line.

310. The angle ACB of any triangle is bisected, and the base AB is bisected at right angles, by straight lines which intersect at D: shew that the angles ACB, ADB are together equal to two right angles.

311. $ACDB$ is a semicircle, AB being the diameter, and the two chords AD, BC intersect at E: shew that if a circle be described round CDE it will cut the former at right angles.

312. The diagonals of a given quadrilateral $ABCD$ intersect at O: shew that the centres of the circles described about the triangles OAB, OBC, OCD, ODA, will lie in the angular points of a parallelogram.

313. A circle is described round the triangle ABC; the tangent at C meets AB produced at D; the circle whose centre is D and radius DC cuts AB at E: shew that EC bisects the angle ACB.

314. AB, AC are two straight lines given in position; BC is a straight line of given length; D, E are the middle points of AB, AC; DF, EF are drawn at right angles to AB, AC respectively. Shew that AF will be constant for all positions of BC.

315. A circle is described about an isosceles triangle ABC in which AB is equal to AC; from A a straight line is drawn meeting the base at D and the circle at E: shew that the circle which passes through B, D, and E, touches AB.

316. AC is a chord of a given circle; B and D are two given points in the chord, both within or both without the circle: if a circle be described to pass through B and D, and touch the given circle, shew that AB and CD subtend equal angles at the point of contact.

317. A and B are two points within a circle: find the point P in the circumference such that if PAH, PBK be drawn meeting the circle at H and K, the chord HK shall be the greatest possible.

318. The centre of a given circle is equidistant from two given straight lines: describe another circle which shall touch these two straight lines and shall cut off from the given circle a segment containing an angle equal to a given angle.

319. O is the centre of the circle circumscribing a triangle ABC; D, E, F the feet of the perpendiculars from A, B, C on the opposite sides: shew that OA, OB, OC are respectively perpendicular to EF, FD, DE.

320. If from any point in the circumference of a given circle straight lines be drawn to the four angular points of an inscribed square, the sum of the squares on the four straight lines is double the square on the diameter.

321. Shew that no rectangle except a square can be described about a circle.

322. Describe a circle about a given rectangle.

323. If tangents be drawn through the extremities of two diameters of a circle the parallelogram thus formed will be a rhombus.

IV. 10.

324. Shew that the angle ACD in the figure of IV. 10 is equal to three times the angle at the vertex of the triangle.

325. Shew that in the figure of IV. 10 there are two triangles which possess the required property: shew that there is also an isosceles triangle whose equal angles are each one third part of the third angle.

326. Shew that the base of the triangle in IV. 10 is equal to the side of a regular pentagon inscribed in the smaller circle of the figure.

327. On a given straight line as base describe an isosceles triangle having the third angle treble of each of the angles at the base.

328. In the figure of IV. 10 suppose the two circles to cut again at E: then DE is equal to DC.

329. If A be the vertex and BD the base of the constructed triangle in IV. 10, D being one of the two points of intersection of the two circles employed in the construction, and E the other, and AE be drawn meeting BD produced at G, shew that GAB is another isosceles triangle of the same kind.

330. In the figure of IV. 10 if the two equal chords of the smaller circle be produced to cut the larger, and these points of section be joined, another triangle will be formed having the property required by the proposition.

331. In the figure of IV. 10 suppose the two circles to cut again at E; join AE, CE, and produce AE, BD to meet at G: then $CDGE$ is a parallelogram.

332. Shew that the smaller of the two circles employed in the figure of IV. 10 is equal to the circle described round the required triangle.

333. In the figure of IV. 10 if AF be the diameter of the smaller circle, DF is equal to a radius of the circle which circumscribes the triangle BCD.

IV. 11 to 16.

334. The straight lines which connect the angular points of a regular pentagon which are not adjacent intersect at the angular points of another regular pentagon.

335. $ABCDE$ is a regular pentagon; join AC and BE, and let BE meet AC at F; shew that AC is equal to the sum of AB and BF.

336. Shew that each of the triangles made by joining the extremities of adjoining sides of a regular pentagon is less than a third and greater than a fourth of the whole area of the pentagon.

337. Shew how to derive a regular hexagon from an equilateral triangle inscribed in a circle, and from the construction shew that the side of the hexagon equals the radius of the circle, and that the hexagon is double of the triangle.

338. In a given circle inscribe a triangle whose angles are as the numbers 2, 5, 8.

339. If $ABCDEF$ is a regular hexagon, and AC, BD, CE, DF, EA, FB be joined, another hexagon is formed whose area is one third of that of the former.

340. Any equilateral figure which is inscribed in a circle is also equiangular.

VI. 1, 2.

341. Shew that one of the triangles in the figure of IV. 10 is a mean proportional between the other two.

342. Through D, any point in the base of a triangle ABC, straight lines DE, DF are drawn parallel to the sides AB, AC, and meeting the sides at E, F: shew that the triangle AEF is a mean proportional between the triangles FBD, EDC.

343. Perpendiculars are drawn from any point within an equilateral triangle on the three sides: shew that their sum is invariable.

344. Find a point within a triangle such that if straight lines be drawn from it to the three angular points the triangle will be divided into three equal triangles.

345. From a point E in the common base of two triangles ACB, ADB, straight lines are drawn parallel to AC, AD, meeting BC, BD at F, G: shew that FG is parallel to CD.

346. From any point in the base of a triangle straight lines are drawn parallel to the sides: shew that the intersection of the diagonals of every parallelogram so formed lies in a certain straight line.

347. In a triangle ABC a straight line AD is drawn perpendicular to the straight line BD which bisects the angle B: shew that a straight line drawn from D parallel to BC will bisect AC.

348. ABC is a triangle; any straight line parallel to BC meets AB at D and AC at E; join BE and CD meeting at F: shew that the triangle ADF is equal to the triangle AEF.

349. ABC is a triangle; any straight line parallel to BC meets AB at D and AC at E; join BE and CD meeting at F: shew that if AF be produced it will bisect BC.

350. If two sides of a quadrilateral figure be parallel to each other, any straight line drawn parallel to them will cut the other sides, or those sides produced, proportionally.

351. ABC is a triangle; it is required to draw from a given point P, in the side AB, or AB produced, a straight line to AC, or AC produced, so that it may be bisected by BC.

VI. 3, A.

352. The side BC of a triangle ABC is bisected at D, and the angles ADB, ADC are bisected by the straight lines DE, DF, meeting AB, AC at E, F respectively: shew that EF is parallel to BC.

353. AB is a diameter of a circle, CD is a chord at right angles to it, and E is any point in CD; AE and BE

are drawn and produced to cut the circle at *F* and *G*: shew that the quadrilateral *CFDG* has any two of its adjacent sides in the same ratio as the remaining two.

354. Apply VI. 3 to solve the problem of the trisection of a finite straight line.

355. In the circumference of the circle of which *AB* is a diameter, take any point *P*; and draw *PC*, *PD* on opposite sides of *AP*, and equally inclined to it, meeting *AB* at *C* and *D*: shew that *AC* is to *BC* as *AD* is to *BD*.

356. *AB* is a straight line, and *D* is any point in it: determine a point *P* in *AB* produced such that *PA* is to *PB* as *DA* is to *DB*.

357. From the same point *A* straight lines are drawn making the angles *BAC*, *CAD*, *DAE* each equal to half a right angle, and they are cut by a straight line *BCDE*, which makes *BAE* an isosceles triangle: shew that *BC* or *DE* is a mean proportional between *BE* and *CD*.

358. The angle *A* of a triangle *ABC* is bisected by *AD* which cuts the base at *D*, and *O* is the middle point of *BC*: shew that *OD* bears the same ratio to *OB* that the difference of the sides bears to their sum.

359. *AD* and *AE* bisect the interior and exterior angles at *A* of a triangle *ABC*, and meet the base at *D* and *E*; and *O* is the middle point of *BC*: shew that *OB* is a mean proportional between *OD* and *OE*.

360. Three points *D*, *E*, *F* in the sides of a triangle *ABC* being joined form a second triangle, such that any two sides make equal angles with the side of the former at which they meet: shew that *AD*, *BE*, *CF* are at right angles to *BC*, *CA*, *AB* respectively.

VI. 4 to 6.

361. If two triangles be on equal bases and between the same parallels, any straight line parallel to their bases will cut off equal areas from the two triangles.

362. *AB* and *CD* are two parallel straight lines; *E* is the middle point of *CD*; *AC* and *BE* meet at *F*, and *AE* and *BD* meet at *G*: shew that *FG* is parallel to *AB*.

363. *A*, *B*, *C* are three fixed points in a straight line; any straight line is drawn through *C*; shew that the perpendiculars on it from *A* and *B* are in a constant ratio.

364. If the perpendiculars from two fixed points on a straight line passing between them be in a given ratio, the straight line must pass through a third fixed point.

365. Find a straight line such that the perpendiculars on it from three given points shall be in a given ratio to each other.

366. Through a given point draw a straight line, so that the parts of it intercepted between that point and perpendiculars drawn to the straight line from two other given points may have a given ratio.

367. A tangent to a circle at the point *A* intersects two parallel tangents at *B*, *C*, the points of contact of which with the circle are *D*, *E* respectively; and *BE*, *CD* intersect at *F*: shew that *AF* is parallel to the tangents *BD*, *CE*.

368. *P* and *Q* are fixed points; *AB* and *CD* are fixed parallel straight lines; any straight line is drawn from *P* to meet *AB* at *M*, and a straight line is drawn from *Q* parallel to *PM* meeting *CD* at *N*: shew that the ratio of *PM* to *QN* is constant, and thence shew that the straight line through *M* and *N* passes through a fixed point.

369. Shew that the diagonals of a quadrilateral, two of whose sides are parallel and one of them double of the other, cut one another at a point of trisection.

370. *A* and *B* are two points on the circumference of a circle of which *C* is the centre; draw tangents at *A* and *B* meeting at *T*; and from *A* draw *AN* perpendicular to *CB*: shew that *BT* is to *BC* as *BN* is to *NA*.

371. In the sides *AB*, *AC* of a triangle *ABC* are taken two points *D*, *E*, such that *BD* is equal to *CE*; *DE*, *BC* are produced to meet at *F*: shew that *AB* is to *AC* as *EF* is to *DF*.

372. If through the vertex and the extremities of the base of a triangle two circles be described intersecting each other in the base or base produced, their diameters are proportional to the sides of the triangle.

373. Find a point the perpendiculars from which on the sides of a given triangle shall be in a given ratio.

374. On *AB*, *AC*, two adjacent sides of a rectangle, two similar triangles are constructed, and perpendiculars are drawn to *AB*, *AC* from the angles which they subtend, intersecting at the point *P*. If *AB*, *AC* be homologous

sides, shew that P is in all cases in one of the diagonals of the rectangle.

375. In the figure of I. 43 shew that if EG and FH be produced they will meet on AC produced.

376. APB and CQD are parallel straight lines, and AP is to PB as DQ is to QC: shew that the straight lines PQ, AC, BD, produced if necessary, will meet at a point: shew also that the straight lines PQ, AD, BC, produced if necessary, will meet at a point.

377. ACB is a triangle, and the side AC is produced to D so that CD is equal to AC, and BD is joined: if any straight line drawn parallel to AB cuts the sides AC, CB, and from the points of section straight lines be drawn parallel to DB, shew that these straight lines will meet AB at points equidistant from its extremities.

378. If a circle be described touching externally two given circles, the straight line passing through the points of contact will intersect the straight line passing through the centres of the given circles at a fixed point.

379. D is the middle point of the side BC of a triangle ABC, and P is any point in AD; through P the straight lines BPE, CPF are drawn meeting the other sides at E, F: shew that EF is parallel to BC.

380. AB is the diameter of a circle, E the middle point of the radius OB; on AE, EB as diameters circles are described; PQL is a common tangent meeting the circles at P and Q, and AB produced at L: shew that BL is equal to the radius of the smaller circle.

381. $ABCDE$ is a regular pentagon, and AD, BE intersect at O: shew that a side of the pentagon is a mean proportional between AO and AD.

382. $ABCD$ is a parallelogram; P and Q are points in a straight line parallel to AB; PA and QB meet at R, and PD and QC meet at S; shew that RS is parallel to AD.

383. A and B are two given points; AC and BD are perpendicular to a given straight line CD; AD and BC intersect at E, and EF is perpendicular to CD: shew that AF and BF make equal angles with CD.

384. From the angular points of a parallelogram $ABCD$ perpendiculars are drawn on the diagonals meeting them at E, F, G, H respectively: shew that $EFGH$ is a parallelogram similar to $ABCD$.

385. If at a given point two circles intersect, and their centres lie on two fixed straight lines which pass through that point, shew that whatever be the magnitude of the circles their common tangents will always meet in one of two fixed straight lines which pass through the given point.

VI. 7 to 18.

386. If two circles touch each other, and also touch a given straight line, the part of the straight line between the points of contact is a mean proportional between the diameters of the circles.

387. Divide a given arc of a circle into two parts, so that the chords of these parts shall be to each other in a given ratio.

388. In a given triangle draw a straight line parallel to one of the sides, so that it may be a mean proportional between the segments of the base.

389. *ABC* is a triangle, and a perpendicular is drawn from *A* to the opposite side, meeting it at *D* between *B* and *C*: shew that if *AD* is a mean proportional between *BD* and *CD* the angle *BAC* is a right angle.

390. *ABC* is a triangle. and a perpendicular is drawn from *A* on the opposite side, meeting it at *D* between *B* and *C*: shew that if *BA* is a mean proportional between *BD* and *BC*, the angle *BAC* is a right angle.

391. *C* is the centre of a circle, and *A* any point within it; *CA* is produced through *A* to a point *B* such that the radius is a mean proportional between *CA* and *CB*: shew that if *P* be any point on the circumference, the angles *CPA* and *CBP* are equal.

392. *O* is a fixed point in a given straight line *OA*, and a circle of given radius moves so as always to be touched by *OA*; a tangent *OP* is drawn from *O* to the circle, and in *OP* produced *PQ* is taken a third proportional to *OP* and the radius: shew that as the circle moves along *OA*, the point *Q* will move in a straight line.

393. Two given parallel straight lines touch a circle, and *SPT* is another tangent cutting the two former tangents at *S* and *T*, and meeting the circle at *P*: shew

that the rectangle SP, PT is constant for all positions of P.

394. Find a point in a side of a triangle, from which two straight lines drawn, one to the opposite angle, and the other parallel to the base, shall cut off towards the vertex and towards the base, equal triangles.

395. ACB is a triangle having a right angle at C; from A a straight line is drawn at right angles to AB, cutting BC produced at E; from B a straight line is drawn at right angles to AB, cutting AC produced at D: shew that the triangle ECD is equal to the triangle ACB.

396. The straight line bisecting the angle ABC of the triangle ABC meets the straight lines drawn through A and C, parallel to BC and AB respectively, at E and F: shew that the triangles CBE, ABF are equal.

397. Shew that the diagonals of any quadrilateral figure inscribed in a circle divide the quadrilateral into four triangles which are similar two and two; and deduce the theorem of III. 35.

398. AB, CD are any two chords of a circle passing through a point O; EF is any chord parallel to OB; join CE, DF meeting AB at the points G and H, and DE, CF meeting AB at the points K and L: shew that the rectangle OG, OH is equal to the rectangle OK, OL.

399. $ABCD$ is a quadrilateral in a circle; the straight lines CE, DE which bisect the angles ACB, ADB cut BD and AC at F and G respectively: shew that EF is to EG as ED is to EC.

400. From an angle of a triangle two straight lines are drawn, one to any point in the side opposite to the angle, and the other to the circumference of the circumscribing circle, so as to cut from it a segment containing an angle equal to the angle contained by the first drawn line and the side which it meets: shew that the rectangle contained by the sides of the triangle is equal to the rectangle contained by the straight lines thus drawn.

401. The vertical angle C of a triangle is bisected by a straight line which meets the base at D, and is produced to a point E, such that the rectangle contained by CD and CE is equal to the rectangle contained by AC and CB: shew that if the base and vertical angle be given, the position of E is invariable.

402. A square is inscribed in a right-angled triangle *ABC*, one side *DE* of the square coinciding with the hypotenuse *AB* of the triangle: shew that the area of the square is equal to the rectangle *AD*, *BE*.

403. *ABCD* is a parallelogram; from *B* a straight line is drawn cutting the diagonal *AC* at *F*, the side *DC* at *G*, and the side *AD* produced at *E*: shew that the rectangle *EF*, *FG* is equal to the square on *BF*.

404. If a straight line drawn from the vertex of an isosceles triangle to the base, be produced to meet the circumference of a circle described about the triangle, the rectangle contained by the whole line so produced, and the part of it between the vertex and the base shall be equal to the square on either of the equal sides of the triangle.

405. Two straight lines are drawn from a point *A* to touch a circle of which the centre is *E*; the points of contact are joined by a straight line which cuts *EA* at *H*; and on *HA* as diameter a circle is described: shew that the straight lines drawn through *E* to touch this circle will meet it on the circumference of the given circle.

VI. 19 to *D*.

406. An isosceles triangle is described having each of the angles at the base double of the third angle: if the angles at the base be bisected, and the points where the lines bisecting them meet the opposite sides be joined, the triangle will be divided into two parts in the proportion of the base to the side of the triangle.

407. Any regular polygon inscribed in a circle is a mean proportional between the inscribed and circumscribed regular polygons of half the number of sides.

408. In the figure of VI. 24 shew that *EG* and *KH* are parallel.

409. Divide a triangle into two equal parts by a straight line at right angles to one of the sides.

410. Through any point *P* in the diagonal *AC* of a parallelogram *ABCD* a straight line is drawn meeting *BC* at *E*, and *AD* at *F*; and through *P* another straight line is drawn meeting *AB* at *G*, and *CD* at *H*: shew that *GF* is parallel to *EH*.

411. Through a given point draw a chord in a given circle so that it shall be divided at the point in a given ratio.

412. From a point without a circle draw a straight line cutting the circle, so that the two segments shall be equal to each other.

413. In the figure of II. 11 shew that four other straight lines, besides the given straight line are divided in the required manner.

414. Construct a triangle, having given the base, the vertical angle, and the rectangle contained by the sides.

415. A circle is described round an equilateral triangle, and from any point in the circumference straight lines are drawn to the angular points of the triangle: shew that one of these straight lines is equal to the other two together.

416. From the extremities *B*, *C* of the base of an isosceles triangle *ABC*, straight lines are drawn at right angles to *AB*, *AC* respectively, and intersecting at *D*: shew that the rectangle *BC*, *AD* is double of the rectangle *AB*, *DB*.

417. *ABC* is an isosceles triangle, the side *AB* being equal to *AC*; *F* is the middle point of *BC*; on any straight line through *A* perpendiculars *FG* and *CE* are drawn: shew that the rectangle *AC*, *EF* is equal to the sum of the rectangles *FC*, *EG* and *FA*, *FG*.

XI. 1 to 12.

418. Shew that equal straight lines drawn from a given point to a given plane are equally inclined to the plane.

419. If two straight lines in one plane be equally inclined to another plane, they will be equally inclined to the common section of these planes.

420. From a point *A* a perpendicular is drawn to a plane meeting it at *B*; from *B* a perpendicular is drawn on a straight line in the plane meeting it at *C*: shew that *AC* is perpendicular to the straight line in the plane.

421. *ABC* is a triangle; the perpendiculars from *A* and *B* on the opposite sides meet at *D*; through *D* a straight line is drawn perpendicular to the plane of the triangle, and *E* is any point in this straight line: shew that

the straight line joining E to any angular point of the triangle is at right angles to the straight line drawn through that angular point parallel to the opposite side of the triangle.

422. Straight lines are drawn from two given points without a given plane meeting each other in that plane: find when their sum is the least possible.

423. Three straight lines not in the same plane meet at a point, and a plane cuts these straight lines at equal distances from the point of intersection: shew that the perpendicular from that point on the plane will meet it at the centre of the circle described about the triangle formed by the portion of the plane intercepted by the planes passing through the straight lines.

424. Give a geometrical construction for drawing a straight line which shall be equally inclined to three straight lines meeting at a point.

425. From a point E draw EC, ED perpendicular to two planes CAB, DAB which intersect in AB, and from D draw DF perpendicular to the plane CAB meeting it at F: shew that the straight line CF, produced if necessary, is perpendicular to AB.

426. Perpendiculars are drawn from a point to a plane, and to a straight line in that plane: shew that the straight line joining the feet of the perpendiculars is perpendicular to the former straight line.

XI. 13 to 21.

427. BCD is the common base of two pyramids, whose vertices A and E lie in a plane passing through BC; and AB, AC are respectively perpendicular to the faces BED, CED: shew that one of the angles at A together with the angles at E make up four right angles.

428. Within the area of a given triangle is inscribed another triangle: shew that the sum of the angles subtended by the sides of the interior triangle at any point not in the plane of the triangles is less than the sum of the angles subtended at the same point by the sides of the exterior angle.

429. From the extremities of the two parallel straight

lines AB, CD parallel straight lines Aa, Bb, Cc, Dd are drawn meeting a plane at a, b, c, d: shew that AB is to CD as ab to cd.

430. Shew that the perpendicular drawn from the vertex of a regular tetrahedron on the opposite face is three times that drawn from its own foot on any of the other faces.

431. A triangular pyramid stands on an equilateral base and the angles at the vertex are right angles: shew that the sum of the perpendiculars on the faces from any point of the base is constant.

432. Three straight lines not in the same plane intersect at a point, and through their point of intersection another straight line is drawn within the solid angle formed by them: shew that the angles which this straight line makes with the first three are together less than the sum, but greater than half the sum, of the angles which the first three make with each other.

433. Three straight lines which do not all lie in one plane, are cut in the same ratio by three planes, two of which are parallel: shew that the third will be parallel to the other two, if its intersections with the three straight lines are not all in the same straight line.

434. Draw two parallel planes, one through one straight line, and the other through another straight line which does not meet the former.

435. If two planes which are not parallel be cut by two parallel planes, the lines of section of the first two by the last two will contain equal angles.

436. From a point A in one of two planes are drawn AB at right angles to the first plane, and AC perpendicular to the second plane, and meeting the second plane at B, C: shew that BC is perpendicular to the line of intersection of the two planes.

437. Polygons formed by cutting a prism by parallel planes are equal.

438. Polygons formed by cutting a pyramid by parallel planes are similar.

439. The straight line $PBbp$ cuts two parallel planes at B, b, and the points P, p are equidistant from the planes; PAa, pcC are other straight lines drawn from P, p to cut the planes: shew that the triangles ABC, abc are equal.

440. Perpendiculars AE, BF are drawn to a plane

from two points A, B above it; a plane is drawn through A perpendicular to AB: shew that its line of intersection with the given plane is perpendicular to EF.

I. 1 to 48.

441. ABC is a triangle, and P is any point within it: shew that the sum of PA, PB, PC is less than the sum of the sides of the triangle.

442. From the centres A and B of two circles parallel radii AP, BQ are drawn; the straight line PQ meets the circumferences again at R and S: shew that AR is parallel to BS.

443. If any point be taken within a parallelogram the sum of the triangles formed by joining the point with the extremities of a pair of opposite sides is half the parallelogram.

444. If a quadrilateral figure be bisected by one diagonal the second diagonal is bisected by the first.

445. Any quadrilateral figure which is bisected by both of its diagonals is a parallelogram.

446. In the figure of I. 5 if the equal sides of the triangle be produced upwards through the vertex, instead of downwards through the base, a demonstration of I. 15 may be obtained without assuming any proposition beyond I. 5.

447. A is a given point, and B is a given point in a given straight line: it is required to draw from A to the given straight line, a straight line AP, such that the sum of AP and PB may be equal to a given length.

448. Shew that by superposition the first case of I. 26 may be immediately demonstrated, and also the second case with the aid of I. 16.

449. A straight line is drawn terminated by one of the sides of an isosceles triangle, and by the other side produced, and bisected by the base: shew that the straight lines thus intercepted between the vertex of the isosceles triangle and this straight line, are together equal to the two equal sides of the triangle.

450. Through the middle point M of the base BC of a triangle a straight line DME is drawn, so as to cut off equal parts from the sides AB, AC, produced if necessary: shew that BD is equal to CE.

451. Of all parallelograms which can be formed with diameters of given lengths the rhombus is the greatest.

452. Shew from I. 18 and I. 32 that if the hypotenuse BC of a right-angled triangle ABC be bisected at D, then AD, BD, CD are all equal.

453. If two equal straight lines intersect each other any where at right angles, the quadrilateral formed by joining their extremities is equal to half the square on either straight line.

454. Inscribe a parallelogram in a given triangle, in such a manner that its diagonals shall intersect at a given point within the triangle.

455. Construct a triangle of given area, and having two of its sides of given lengths.

456. Construct a triangle, having given the base, the difference of the sides, and the difference of the angles at the base.

457. AB, AC are two given straight lines: it is required to find in AB a point P, such that if PQ be drawn perpendicular to AC, the sum of AP and AQ may be equal to a given straight line.

458. The distance of the vertex of a triangle from the bisection of its base, is equal to, greater than, or less than half of the base, according as the vertical angle is a right, an acute, or an obtuse angle.

459. If in the sides of a given square, at equal distances from the four angular points, four other points be taken, one on each side, the figure contained by the straight lines which join them, shall also be a square.

460. On a given straight line as base, construct a triangle, having given the difference of the sides and a point through which one of the sides is to pass.

461. *ABC* is a triangle in which *BA* is greater than *CA*; the angle *A* is bisected by a straight line which meets *BC* at *D*; shew that *BD* is greater than *CD*.

462. If one angle of a triangle be triple another the triangle may be divided into two isosceles triangles.

463. If one angle of a triangle be double another, an isosceles triangle may be added to it so as to form together with it a single isosceles triangle.

464. Let one of the equal sides of an isosceles triangle be bisected at *D*, and let it also be doubled by being produced through the extremity of the base to *E*, then the distance of the other extremity of the base from *E* is double its distance from *D*.

465. Determine the locus of a point whose distance from one given point is double its distance from another given point.

466. A straight line *AB* is bisected at *C*, and on *AC* and *CB* as diagonals any two parallelograms *ADCE* and *CFBG* are described; let the parallelogram whose adjacent sides are *CD* and *CF* be completed, and also that whose adjacent sides are *CE* and *CG*: shew that the diagonals of these latter parallelograms are in the same straight line.

467. *ABCD* is a rectangle of which *A*, *C* are opposite angles; *E* is any point in *BC* and *F* is any point in *CD*: shew that twice the area of the triangle *AEF*, together with the rectangle *BE*, *DF* is equal to the rectangle *ABCD*.

468. *ABC*, *DBC* are two triangles on the same base, and *ABC* has the side *AB* equal to the side *AC*; a circle passing through *C* and *D* has its centre *E* on *CA*, produced if necessary; a circle passing through *B* and *D* has its centre *F* on *BA*, produced if necessary: shew that the quadrilateral *AEDF* has the sum of two of its sides equal to the sum of the other two.

469. Two straight lines *AB*, *AC* are given in position:

it is required to find in AB a point P, such that a perpendicular being drawn from it to AC, the straight line AP may exceed this perpendicular by a proposed length.

470. Shew that the opposite sides of any equiangular hexagon are parallel, and that any two sides which are adjacent are together equal to the two to which they are parallel.

471. From D and E, the corners of the square $BDEC$ described on the hypotenuse BC of a right-angled triangle ABC, perpendiculars DM, EN are let fall on AC, AB respectively: shew that AM is equal to AB, and AN equal to AC.

472. AB and AC are two given straight lines, and P is a given point: it is required to draw through P a straight line which shall form with AB and AC the least possible triangle.

473. ABC is a triangle in which C is a right angle: shew how to draw a straight line parallel to a given straight line, so as to be terminated by CA and CB, and bisected by AB.

474. ABC is an isosceles triangle having the angle at B four times either of the other angles; AB is produced to D so that BD is equal to twice AB, and CD is joined: shew that the triangles ACD and ABC are equiangular to one another.

475. Through a point K within a parallelogram $ABCD$ straight lines are drawn parallel to the sides: shew that the difference of the parallelograms of which KA and KC are diagonals is equal to twice the triangle BKD.

476. Construct a right-angled triangle, having given one side and the difference between the other side and the hypotenuse.

477. The straight lines AD, BE bisecting the sides BC, AC of a triangle intersect at G: shew that AG is double of GD.

478. BAC is a right-angled triangle; one straight line is drawn bisecting the right angle A, and another bisecting the base BC at right angles; these straight lines intersect at E: if D be the middle point of BC, shew that DE is equal to DA.

479. On AC the diagonal of a square $ABCD$, a rhombus $AEFC$ is described of the same area as the square,

and having its acute angle at *A*: if *AF* be joined, shew that the angle *BAC* is divided into three equal angles.

480. *AB*, *AC* are two fixed straight lines at right angles; *D* is any point in *AB*, and *E* is any point in *AC*; on *DE* as diagonal a half square is described with its vertex at *G*: shew that the locus of *G* is the straight line which bisects the angle *BAC*.

481. Shew that a square is greater than any other parallelogram of the same perimeter.

482. Inscribe a square of given magnitude in a given square.

483. *ABC* is a triangle; *AD* is a third of *AB*, and *AE* is a third of *AC*; *CD* and *BE* intersect at *F*: shew that the triangle *BFC* is half the triangle *BAC*, and that the quadrilateral *ADFE* is equal to either of the triangles *CFE* or *BDF*.

484. *ABC* is a triangle, having the angle *C* a right angle; the angle *A* is bisected by a straight line which meets *BC* at *D*, and the angle *B* is bisected by a straight line which meets *AC* at *E*; *AD* and *BE* intersect at *O*: shew that the triangle *AOB* is half the quadrilateral *ABDE*.

485. Shew that a scalene triangle cannot be divided by a straight line into two parts which will coincide.

486. *ABCD*, *ACED* are parallelograms on equal bases *BC*, *CE*, and between the same parallels *AD*, *BE*; the straight lines *BD* and *AE* intersect at *F*: shew that *BF* is equal to twice *DF*.

487. Parallelograms *AFGC*, *CBKH* are described on *AC*, *BC* outside the triangle *ABC*; *FG* and *KH* meet at *Z*; *ZC* is joined, and through *A* and *B* straight lines *AD* and *BE* are drawn, both parallel to *ZC*, and meeting *FG* and *KH* at *D* and *E* respectively: shew that the figure *ADEB* is a parallelogram, and that it is equal to the sum of the parallelograms *FC*, *CK*.

488. If a quadrilateral have two of its sides parallel shew that the straight line drawn parallel to these sides through the intersection of the diagonals is bisected at that point.

489. Two triangles are on equal bases and between the same parallels: shew that the sides of the triangles intercept equal lengths of any straight line which is parallel to their bases.

490. In a right-angled triangle, right-angled at A, if the side AC be double of the side AB, the angle B is more than double of the angle C.

491. Trisect a parallelogram by straight lines drawn through one of its angular points.

492. AHK is an equilateral triangle; $ABCD$ is a rhombus, a side of which is equal to a side of the triangle, and the sides BC and CD of which pass through H and K respectively: shew that the angle A of the rhombus is ten-ninths of a right angle.

493. Trisect a given triangle by straight lines drawn from a given point in one of its sides.

494. In the figure of I. 35 if two diagonals be drawn to the two parallelograms respectively, one from each extremity of the base, and the intersection of the diagonals be joined with the intersection of the sides (or sides produced) in the figure, shew that the joining straight line will bisect the base.

II. 1 to 14.

495. Produce one side of a given triangle so that the rectangle contained by this side and the produced part may be equal to the difference of the squares on the other two sides.

496. Produce a given straight line so that the sum of the squares on the given straight line and on the part produced may be equal to twice the rectangle contained by the whole straight line thus produced and the part produced.

497. Produce a given straight line so that the sum of the squares on the given straight line and on the whole straight line thus produced may be equal to twice the rectangle contained by the whole straight line thus produced and the part produced.

498. Produce a given straight line so that the rectangle contained by the whole straight line thus produced and the part produced may be equal to the square on the given straight line.

499. Describe an isosceles obtuse-angled triangle such that the square on the largest side may be equal to three times the square on either of the equal sides.

500. Find the obtuse angle of a triangle when the

square on the side opposite to the obtuse angle is greater than the sum of the squares on the sides containing it, by the rectangle of the sides.

501. Construct a rectangle equal to a given square when the sum of two adjacent sides of the rectangle is equal to a given quantity.

502. Construct a rectangle equal to a given square when the difference of two adjacent sides of the rectangle is equal to a given quantity.

503. The least square which can be inscribed in a given square is that which is half of the given square.

504. Divide a given straight line into two parts so that the squares on the whole line and on one of the parts may be together double of the square on the other part.

505. Two rectangles have equal areas and equal perimeters: shew that they are equal in all respects.

506. *ABCD* is a rectangle; *P* is a point such that the sum of *PA* and *PC* is equal to the sum of *PB* and *PD*: shew that the locus of *P* consists of the two straight lines through the centre of the rectangle parallel to its sides.

III. 1 to 37.

507. Describe a circle which shall pass through a given point and touch a given straight line at a given point.

508. Describe a circle which shall pass through a given point and touch a given circle at a given point.

509. Describe a circle which shall touch a given circle at a given point and touch a given straight line.

510. *AD*, *BE* are perpendiculars from the angles *A* and *B* of a triangle on the opposite sides; *BF* is perpendicular to *ED* or *ED* produced: shew that the angle *FBD* is equal to the angle *EBA*.

511. If *ABC* be a triangle, and *BE*, *CF* the perpendiculars from the angles on the opposite sides, and *K* the middle point of the third side, shew that the angles *FEK*, *EFK* are each equal to *A*.

512. *AB* is a diameter of a circle; *AC* and *AD* are two chords meeting the tangent at *B* at *E* and *F* respectively: shew that the angles *FCE* and *FDE* are equal.

513. Shew that the four straight lines bisecting the angles of any quadrilateral form a quadrilateral which can be inscribed in a circle.

514. Find the shortest distance between two circles which do not meet.

515. Two circles cut one another at a point *A*: it is required to draw through *A* a straight line so that the extreme length of it intercepted by the two circles may be equal to that of a given straight line.

516. If a polygon of an even number of sides be inscribed in a circle, the sum of the alternate angles together with two right angles is equal to as many right angles as the figure has sides.

517. Draw from a given point in the circumference of a circle, a chord which shall be bisected by its point of intersection with a given chord of the circle.

518. When an equilateral polygon is described about a circle it must necessarily be equiangular if the number of sides be odd, but not otherwise.

519. *AB* is the diameter of a circle whose centre is *C*, and *DCE* is a sector having the arc *DE* constant; join *AE*, *BD* intersecting at *P*; shew that the angle *APB* is constant.

520. If any number of triangles on the same base *BC*, and on the same side of it have their vertical angles equal, and perpendiculars, intersecting at *D*, be drawn from *B* and *C* on the opposite sides, find the locus of *D*; and shew that all the straight lines which bisect the angle *BDC* pass through the same point.

521. Let *O* and *C* be any fixed points on the circumference of a circle, and *OA* any chord; then if *AC* be joined and produced to *B*, so that *OB* is equal to *OA*, the locus of *B* is an equal circle.

522. From any point *P* in the diagonal *BD* of a parallelogram *ABCD*, straight lines *PE*, *PF*, *PG*, *PH* are drawn perpendicular to the sides *AB*, *BC*, *CD*, *DA*: shew that *EF* is parallel to *GH*.

523. Through any fixed point of a chord of a circle other chords are drawn; shew that the straight lines from the middle point of the first chord to the middle points of the others will meet them all at the same angle.

524. *ABC* is a straight line, divided at any point *B*,

into two parts; *ADB* and *CDB* are similar segments of circles, having the common chord *BD*; *CD* and *AD* are produced to meet the circumferences at *F* and *E* respectively, and *AF*, *CE*, *BF*, *BE* are joined: shew that *ABF* and *CBE* are isosceles triangles, equiangular to one another.

525. If the centres of two circles which touch each other externally be fixed, the common tangent of those circles will touch another circle of which the straight line joining the fixed centres is the diameter.

526. *A* is a given point: it is required to draw from *A* two straight lines which shall contain a given angle and intercept on a given straight line a part of given length.

527. A straight line and two circles are given: find the point in the straight line from which the tangents drawn to the circles are of equal length.

528. In a circle two chords of given length are drawn so as not to intersect, and one of them is fixed in position; the opposite extremities of the chords are joined by straight lines intersecting within the circle: shew that the locus of the point of intersection will be a portion of the circumference of a circle, passing through the extremities of the fixed chord.

529. *A* and *B* are the centres of two circles which touch internally at *C*, and also touch a third circle, whose centre is *D*, externally and internally respectively at *E* and *F*: shew that the angle *ADB* is double of the angle *ECF*.

530. *C* is the centre of a circle, and *CP* is a perpendicular on a chord *APB*: shew that the sum of *CP* and *AP* is greatest when *CP* is equal to *AP*.

531. *AB*, *BC*, *CD* are three adjacent sides of any polygon inscribed in a circle; the arcs *AB*, *BC*, *CD* are bisected at *L*, *M*, *N*; and *LM* cuts *BA*, *BC* respectively at *P* and *Q*: shew that *BPQ* is an isosceles triangle; and that the angles *ABC*, *BCD* are together double of the angle *LMN*.

532. In the circumference of a given circle determine a point so situated that if chords be drawn to it from the extremities of a given chord of the circle their difference shall be equal to a given straight line less than the given chord.

533. Construct a triangle, having given the sum of the

sides, the difference of the segments of the base made by the perpendicular from the vertex, and the difference of the base angles.

534. On a straight line AB as base, and on the same side of it are described two segments of circles; P is any point in the circumference of one of the segments, and the straight line BP cuts the circumference of the other segment at Q: shew that the angle PAQ is equal to the angle between the tangents at A.

535. AKL is a fixed straight line cutting a given circle at K and L; APQ, ARS are two other straight lines making equal angles with AKL, and cutting the circle at P, Q and R, S: shew that whatever be the position of APQ and ARS, the straight line joining the middle points of PQ and RS always remains parallel to itself.

536. If about a quadrilateral another quadrilateral can be described such that every two of its adjacent sides are equally inclined to that side of the former quadrilateral which meets them both, then a circle may be described about the former quadrilateral.

537. Two circles touch one another internally at the point A: it is required to draw from A a straight line such that the part of it between the circles may be equal to a given straight line, which is not greater than the difference between the diameters of the circles.

538. $ABCD$ is a parallelogram; AE is at right angles to AB, and CE is at right angles to CB: shew that ED, if produced, will cut AC at right angles.

539. From each angular point of a triangle a perpendicular is let fall on the opposite side: shew that the rectangles contained by the segments into which each perpendicular is divided by the point of intersection of the three are equal to each other.

540. The two angles at the base of a triangle are bisected by two straight lines on which perpendiculars are drawn from the vertex: shew that the straight line which passes through the feet of these perpendiculars will be parallel to the base and will bisect the sides.

541. In a given circle inscribe a rectangle equal to a given rectilineal figure.

542. In an acute-angled triangle ABC perpendiculars AD, BE are let fall on BC, CA respectively; circles

described on AC, BC as diameters meet BE, AD respectively at F, G and H, K: shew that F, G, H, K lie on the circumference of a circle.

543. Two diameters in a circle are at right angles; from their extremities four parallel straight lines are drawn; shew that they divide the circumference into four equal parts.

544. E is the middle point of a semicircular arc AEB, and CDE is any chord cutting the diameter at D, and the circle at C: shew that the square on CE is twice the quadrilateral $AEBC$.

545. AB is a fixed chord of a circle, AC is a moveable chord of the same circle; a parallelogram is described of which AB and AC are adjacent sides: find the locus of the middle points of the diagonals of the parallelogram.

546. AB is a fixed chord of a circle, AC is a moveable chord of the same circle; a parallelogram is described of which AB and AC are adjacent sides: determine the greatest possible length of the diagonal drawn through A.

547. If two equal circles be placed at such a distance apart that the tangent drawn to either of them from the centre of the other is equal to a diameter, shew that they will have a common tangent equal to the radius.

548. Find a point in a given circle from which if two tangents be drawn to an equal circle, given in position, the chord joining the points of contact is equal to the chord of the first circle formed by joining the points of intersection of the two tangents produced; and determine the limit to the possibility of the problem.

549. AB is a diameter of a circle, and AF is any chord; C is any point in AB, and through C a straight line is drawn at right angles to AB, meeting AF, produced if necessary at G, and meeting the circumference at D: shew that the rectangle FA, AG, and the rectangle BA, AC, and the square on AD are all equal.

550. Construct a triangle, having given the base, the vertical angle, and the length of the straight line drawn from the vertex to the base bisecting the vertical angle.

551. A, B, C are three given points in the circumference of a given circle: find a point P such that if AP, BP, CP meet the circumference at D, E, F respectively, the arcs DE, EF may be equal to given arcs.

552. Find the point in the circumference of a given circle, the sum of whose distances from two given straight lines at right angles to each other, which do not cut the circle, is the greatest or least possible.

553. On the sides of a triangle segments of a circle are described *internally*, each containing an angle equal to the excess of two right angles above the opposite angle of the triangle: shew that the radii of the circles are equal, that the circles all pass through one point, and that their chords of intersection are respectively perpendicular to the opposite sides of the triangle.

IV. 1 to 16.

554. From the angles of a triangle ABC perpendiculars are drawn to the opposite sides meeting them at D, E, F respectively: shew that DE and DF are equally inclined to AD.

555. The points of contact of the inscribed circle of a triangle are joined; and from the angular points of the triangle so formed perpendiculars are drawn to the opposite sides: shew that the triangle of which the feet of these perpendiculars are the angular points has its sides parallel to the sides of the original triangle.

556. Construct a triangle having given an angle and the radii of the inscribed and circumscribed circles.

557. Triangles are constructed on the same base with equal vertical angles; shew that the locus of the centres of the escribed circles, each of which touches one of the sides externally and the other side and base produced, is an arc of a circle, the centre of which is on the circumference of the circle circumscribing the triangles.

558. From the angular points A, B, C of a triangle perpendiculars are drawn on the opposite sides, and terminated at the points D, E, F on the circumference of the circumscribing circle: if L be the point of intersection of the perpendiculars, shew that LD, LE, LF are bisected by the sides of the triangle.

559. *ABCDE* is a regular pentagon; join *AC* and *BD* intersecting at *O*: shew that *AO* is equal to *DO*, and that the rectangle *AC*, *CO* is equal to the square on *BC*.

560. A straight line *PQ* of given length moves so that its ends are always on two fixed straight lines *CP*, *CQ*; straight lines from *P* and *Q* at right angles to *CP* and *CQ* respectively intersect at *R*; perpendiculars from *P* and *Q* on *CQ* and *CP* respectively intersect at *S*: shew that the loci of *R* and *S* are circles having their common centre at *C*.

561. Right-angled triangles are described on the same hypotenuse: shew that the locus of the centres of the inscribed circles is a quarter of the circumference of a circle of which the common hypotenuse is a chord.

562. On a given straight line *AB* any triangle *ACB* is described; the sides *AC*, *BC* are bisected and straight lines drawn at right angles to them through the points of bisection to intersect at a point *D*; find the locus of *D*.

563. Construct a triangle, having given its base, one of the angles at the base, and the distance between the centre of the inscribed circle and the centre of the circle touching the base and the sides produced.

564. Describe a circle which shall touch a given straight line at a given point, and bisect the circumference of a given circle.

565. Describe a circle which shall pass through a given point and bisect the circumferences of two given circles.

566. Within a given circle inscribe three equal circles, touching one another and the given circle.

567. If the radius of a circle be cut as in II. 11, the greater segment will be the side of a regular decagon inscribed in the circle.

568. If the radius of a circle be cut as in II. 11, the square on its greater segment, together with the square on the radius, is equal to the square on the side of a regular pentagon inscribed in the circle.

569. From the vertex of a triangle draw a straight line to the base so that the square on the straight line may be equal to the rectangle contained by the segments of the base.

570. Four straight lines are drawn in a plane forming four triangles; shew that the circumscribing circles of these triangles all pass through a common point.

571. The perpendiculars from the angles A and B of a triangle on the opposite sides meet at D; the circles described round ADC and DBC cut AB or AB produced at the points E and F: shew that AE is equal to BF.

572. The four circles each of which passes through the centres of three of the four circles touching the sides of a triangle are equal to one another.

573. Four circles are described so that each may touch internally three of the sides of a quadrilateral: shew that a circle may be described so as to pass through the centres of the four circles.

574. A circle is described round the triangle ABC, and from any point P of its circumference perpendiculars are drawn to BC, CA, AB, which meet the circle again at D, E, F: shew that the triangles ABC and DEF are equal in all respects, and that the straight lines AD, BE, CF are parallel.

575. With any point in the circumference of a given circle as centre, describe another circle, cutting the former at A and B; from B draw in the described circle a chord BD equal to its radius, and join AD, cutting the given circle at Q: shew that QD is equal to the radius of the given circle.

576. A point is taken without a square, such that straight lines being drawn to the angular points of the square, the angle contained by the two extreme straight lines is divided into three equal parts by the other two straight lines: shew that the locus of the point is the circumference of the circle circumscribing the square.

577. Circles are inscribed in the two triangles formed by drawing a perpendicular from an angle of a triangle on the opposite side; and analogous circles are described in relation to the two other like perpendiculars: shew that the sum of the diameters of the six circles together with the sum of the sides of the original triangle is equal to twice the sum of these perpendiculars.

578. Three concentric circles are drawn in the same plane: draw a straight line, such that one of its segments between the inner and outer circumference may be bisected at one of the points at which the straight line meets the middle circumference.

VI. 1 to D.

579. AB is a diameter, and P any point in the circumference of a circle; AP and BP are joined and produced if necessary; from any point C in AB a straight line is drawn at right angles to AB meeting AP at D and BP at E, and the circumference of the circle at F: shew that CD is a third proportional to CE and CF.

580. A, B, C are three points in a straight line, and D a point at which AB and BC subtend equal angles: shew that the locus of D is the circumference of a circle.

581. If a straight line be drawn from one corner of a square cutting off one-fourth from the diagonal it will cut off one-third from a side. Also if straight lines be drawn similarly from the other corners so as to form a square, this square will be two-fifths of the original square.

582. The sides AB, AC of a given triangle ABC are produced to any points D, E, so that DE is parallel to BC. The straight line DE is divided at F so that DF is to FE as BD is to CE: shew that the locus of F is a straight line.

583. A, B, C are three points in order in a straight line: find a point P in the straight line so that PB may be a mean proportional between PA and PC.

584. A, B are two fixed points on the circumference of a given circle, and P is a moveable point on the circumference; on PB is taken a point D such that PD is to PA in a constant ratio, and on PA is taken a point E such that PE is to PB in the same ratio: shew that DE always touches a fixed circle.

585. ABC is an isosceles triangle, the angle at A being four times either of the others: shew that if BC be bisected at D and E, the triangle ADE is equilateral.

586. Perpendiculars are let fall from two opposite angles of a rectangle on a diagonal: shew that they will divide the diagonal into equal parts, if the square on one side of the rectangle be double that on the other.

587. A straight line AB is divided into any two parts at C, and on the whole straight line and on the two parts of it equilateral triangles ADB, ACE, BCF are described, the two latter being on the same side of the straight

line, and the former on the opposite side; G, H, K are the centres of the circles inscribed in these triangles: shew that the angles AGH, BGK are respectively equal to the angles ADC, BDC, and that GH is equal to GK.

588. On the two sides of a right-angled triangle squares are described: shew that the straight lines joining the acute angles of the triangle and the opposite angles of the squares cut off equal segments from the sides, and that each of these equal segments is a mean proportional between the remaining segments.

589. Two straight lines and a point between them are given in position: draw two straight lines from the given point to terminate in the given straight lines, so that they shall contain a given angle and have a given ratio.

590. With a point A in the circumference of a circle ABC as centre, a circle PBC is described cutting the former circle at the points B and C; any chord AD of the former meets the common chord BC at E, and the circumference of the other circle at O: shew that the angles EPO and DPO are equal for all positions of P.

591. ABC, ABF are triangles on the same base in the ratio of two to one; AF and BF produced meet the sides at D and E; in FB a part FG is cut off equal to FE, and BG is bisected at O: shew that BO is to BE as DF is to DA.

592. A is the centre of a circle, and another circle passes through A and cuts the former at B and C; AD is a chord of the latter circle meeting BC at E, and from D are drawn DF and DG tangents to the former circle: shew that G, E, F lie on one straight line.

593. In AB, AC, two sides of a triangle, are taken points D, E; AB, AC are produced to F, G such that BF is equal to AD, and CG equal to AE; BG, CF are joined meeting at H: shew that the triangle FHG is equal to the triangles BHC, ADE together.

594. In any triangle ABC if BD be taken equal to one-fourth of BC, and CE one-fourth of AC, the straight line drawn from C through the intersection of BE and AD will divide AB into two parts, which are in the ratio of nine to one.

595. Any rectilineal figure is inscribed in a circle: shew that by bisecting the arcs and drawing tangents to the points of bisection parallel to the sides of the recti-

lineal figure, we can form a similar rectilineal figure circumscribing the circle.

596. Find a mean proportional between two similar right-angled triangles which have one of the sides containing the right angle common.

597. In the sides AC, BC of a triangle ABC points D and E are taken, such that CD and CE are respectively the third parts of AC and BC; BD and AE are drawn intersecting at O: shew that EO and DO are respectively the fourth parts of AE and BD.

598. CA, CB are diameters of two circles which touch each other externally at C; a chord AD of the former circle, when produced, touches the latter at E, while a chord BF of the latter, when produced, touches the former at G: shew that the rectangle contained by AD and BF is four times that contained by DE and FG.

599. Two circles intersect at A, and BAC is drawn meeting them at B and C; with B, C as centres are described two circles each of which intersects one of the former at right angles: shew that these circles and the circle whose diameter is BC meet at a point.

600. $ABCDEF$ is a regular hexagon: shew that BF divides AD in the ratio of one to three.

601. ABC, DEF are triangles, having the angle A equal to the angle D; and AB is equal to DF: shew that the areas of the triangles are as AC to DE.

602. If M, N be the points at which the inscribed and an escribed circle touch the side AC of a triangle ABC; shew that if BM be produced to cut the escribed circle again at P, then NP is a diameter.

603. The angle A of a triangle ABC is a right angle, and D is the foot of the perpendicular from A on BC; DM, DN are perpendiculars on AB, AC: shew that the angles BMC, BNC are equal.

604. If from the point of bisection of any given arc of a circle two straight lines be drawn, cutting the chord of the arc and the circumference, the four points of intersection shall also lie in the circumference of a circle.

605. The side AB of a triangle ABC is touched by the inscribed circle at D, and by the escribed circle at E: shew that the rectangle contained by the radii is equal to the rectangle AD, DB and to the rectangle AE, EB.

606. Shew that the locus of the middle points of straight lines parallel to the base of a triangle and terminated by its sides is a straight line.

607. A parallelogram is inscribed in a triangle, having one side on the base of the triangle, and the adjacent sides parallel to a fixed direction: shew that the locus of the intersection of the diagonals of the parallelogram is a straight line bisecting the base of the triangle.

608. On a given straight line AB as hypotenuse a right-angled triangle is described; and from A and B straight lines are drawn to bisect the opposite sides: shew that the locus of their intersection is a circle.

609. From a given point outside two given circles which do not meet, draw a straight line such that the portions of it intercepted by each circle shall be respectively proportional to their radii.

610. In a given triangle inscribe a rhombus which shall have one of its angular points coincident with a point in the base, and a side on that base.

611. ABC is a triangle having a right angle at C; $ABDE$ is the square described on the hypotenuse; F, G, H are the points of intersection of the diagonals of the squares on the hypotenuse and sides: shew that the angles DCE, GFH are together equal to a right angle.

MISCELLANEOUS.

612. O is a fixed point from which any straight line is drawn meeting a fixed straight line at P; in OP a point Q is taken such that the rectangle OP, OQ is constant: shew that the locus of Q is the circumference of a circle.

613. O is a fixed point on the circumference of a circle, from which any straight line is drawn meeting the circumference at P; in OP a point Q is taken such that the rectangle OP, OQ is constant: shew that the locus of Q is a straight line.

614. The opposite sides of a quadrilateral inscribed in a circle when produced meet at P and Q: shew that the square on PQ is equal to the sum of the squares on the tangents from P and Q to the circle.

615. *ABCD* is a quadrilateral inscribed in a circle; the opposite sides *AB* and *DC* are produced to meet at *F*; and the opposite sides *BC* and *AD* at *E*: shew that the circle described on *EF* as diameter cuts the circle *ABCD* at right angles.

616. From the vertex of a right-angled triangle a perpendicular is drawn on the hypotenuse, and from the foot of this perpendicular another is drawn on each side of the triangle: shew that the area of the triangle of which these two latter perpendiculars are two of the sides cannot be greater than one-fourth of the area of the original triangle.

617. If the extremities of two intersecting straight lines be joined so as to form two vertically opposite triangles, the figure made by connecting the points of bisection of the given straight lines, will be a parallelogram equal in area to half the difference of the triangles.

618. *AB*, *AC* are two tangents to a circle, touching it at *B* and *C*; *R* is any point in the straight line which joins the middle points of *AB* and *AC*; shew that *AR* is equal to the tangent drawn from *R* to the circle.

619. *AB*, *AC* are two tangents to a circle; *PQ* is a chord of the circle which, produced if necessary, meets the straight line joining the middle points of *AB*, *AC* at *R*; shew that the angles *RAP*, *AQR* are equal to one another.

620. Shew that the four circles each of which passes through the middle points of the sides of one of the four triangles formed by two adjacent sides and a diagonal of any quadrilateral all intersect at a point.

621. Perpendiculars are drawn from any point on the three straight lines which bisect the angles of an equilateral triangle: shew that one of them is equal to the sum of the other two.

622. Two circles intersect at *A* and *B*, and *CBD* is drawn through *B* perpendicular to *AB* to meet the circles; through *A* a straight line is drawn bisecting either the interior or exterior angle between *AC* and *AD*, and meeting the circumferences at *E* and *F*: shew that the tangents to the circumferences at *E* and *F* will intersect in *AB* produced.

623. Divide a triangle by two straight lines into three

parts, which, when properly arranged, shall form a parallelogram whose angles are of given magnitude.

624. *ABCD* is a parallelogram, and *P* is any point: shew that the triangle *PAC* is equal to the difference of the triangles *PAB* and *PAD*, if *P* is within the angle *BAD* or that which is vertically opposite to it; and that the triangle *PAC* is equal to the sum of the triangles *PAB* and *PAD*, if *P* has any other position.

625. Two circles cut each other, and a straight line *ABCDE* is drawn, which meets one circle at *A* and *D*, the other at *B* and *E*, and their common chord at *C*: shew that the square on *BD* is to the square on *AE* as the rectangle *BC*, *CD* is to the rectangle *AC*, *CE*.

THE END.

CAMBRIDGE: PRINTED AT THE UNIVERSITY PRESS.