❧ SGM1 CORRIGÉ du devoir de mathématiques n°3 ❧

Solution 1

$$f(x,y) = \ln(xy - x^2)$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{y - 2x}{xy - x^2} = \frac{y - 2x}{x(y - x)}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{x}{x(y - x)} = \frac{1}{y - x}$$

Solution 2

$$\frac{\partial f}{\partial x} = -\frac{1}{2} \times 2x \times \frac{1}{(x^2 + y^2)^{3/2}} = -\frac{x}{(x^2 + y^2)\sqrt{x^2 + y^2}}$$

De même $\dfrac{\partial f}{\partial y} = -\dfrac{y}{(x^2 + y^2)\sqrt{x^2 + y^2}}$

Donc $x\dfrac{\partial f}{\partial x}(x,y) + y\dfrac{\partial f}{\partial y}(x,y) = \dfrac{-x^2 - y^2}{(x^2 + y^2)\sqrt{x^2 + y^2}} = -f(x,y)$

Solution 3

$$\frac{1}{R} = \frac{1}{x} + \frac{1}{y} \text{ d'où le résultat.}$$

$$dR = \frac{\partial R}{\partial x}\,dx + \frac{\partial R}{\partial y}\,dy$$

$$\frac{\partial R}{\partial x} = \frac{y(x + y) - xy}{(x + y)^2} = \frac{y^2}{(x + y)^2}$$

$$dR = \frac{y^2}{(x + y)^2}\,dx + \frac{x^2}{(x + y)^2}\,dy$$

Solution 4

$$I = \int_{x=0}^{x=1} x^2 \left(\int_{y=0}^{y=1} \frac{1}{1 + y^2}\,dy \right) dx = \int_{x=0}^{x=1} x^2 \left[\text{Arctan}\,y\right]_{y=0}^{y=1} dx = \frac{\pi}{4}\left[\frac{x^3}{3}\right]_{x=0}^{x=1} = \frac{\pi}{12}$$

Solution 5

$$I_y = \int_{x=0}^{x=1} x^2 \left(\int_{y=0}^{y=-x+1} dy \right) dx = \left[-\frac{x^4}{4} + \frac{x^3}{3}\right]_{x=0}^{x=1} = \frac{1}{12}$$

Solution 6

Passage en polaire.

$$I = \iint_\Delta e^{r^2}\, r\,dr\,d\theta \text{ avec } \Delta = \left\{(r,\theta)\,|\,0 \leqslant r \leqslant 1, 0 \leqslant \theta \leqslant \pi\right\}$$

$$I = \pi \int_{r=0}^{r=1} r\,e^{r^2}\,dr = \frac{\pi}{2}\left[e^{r^2}\right]_{r=0}^{r=1} = \frac{(e - 1)\pi}{2}$$