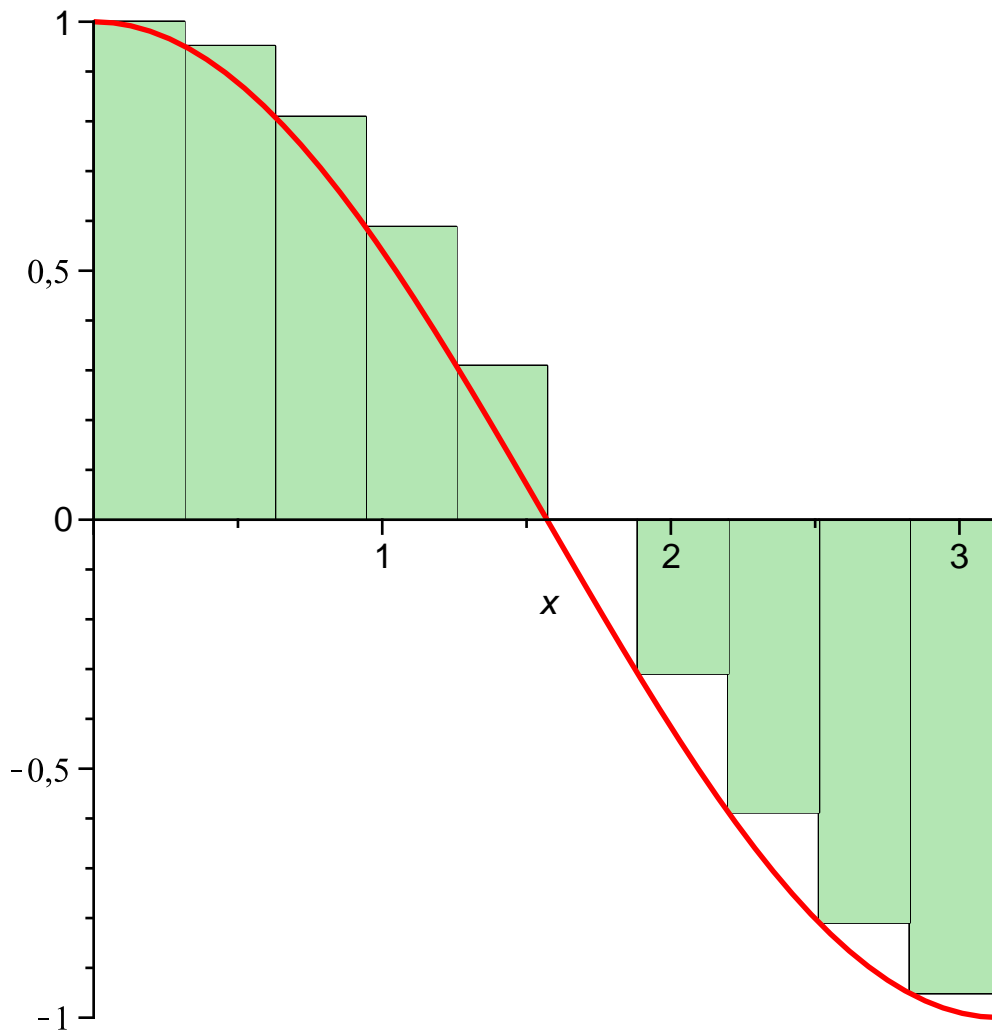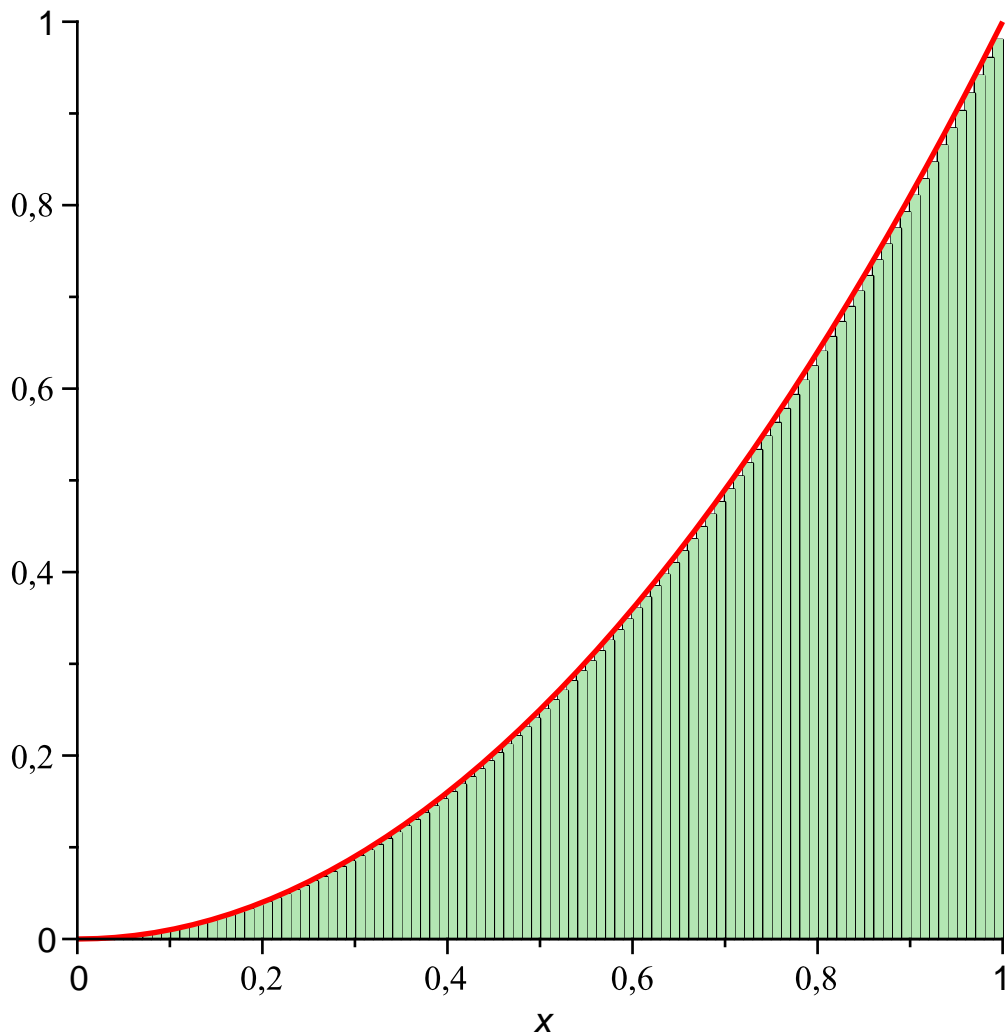```
> Rect:=proc(f,N,a,b)
  local nbrect,i,S,x,n;
  nbrect:=[seq(i,i=1..N)]:
  S:=seq(student[leftbox](f(x),x=a..b,n),n=nbrect):
  plots[display](S,insequence=true);
  end:
```

```
> student[leftbox](cos(x),x=0..Pi,10);
```



```
> Rect(x->x^2,100,0,1);
```

```
> airerect:=proc(f,N,a,b)
  local k:
  Sum((b-a)/N*f(a+k*(b-a)/N),k=1..N)=evalf(sum((b-a)/N*f(a+k*(b-
  a)/N),k=1..N));
  end:
> airerect(x->x^2,1000,0,1);
```
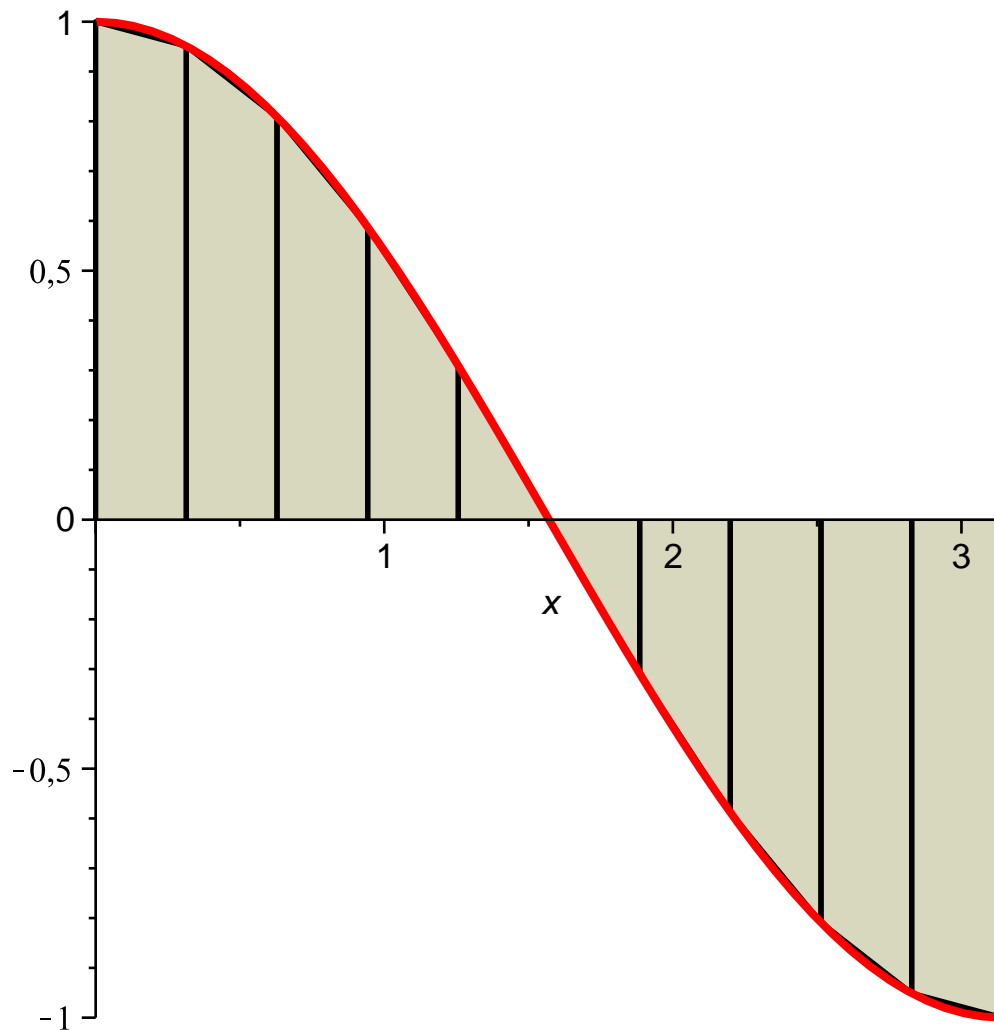
$$\sum_{k=1}^{1000} \frac{1}{1000000000}\, k^2 = 0.3338335000 \tag{1}$$

```
> Trap:=proc(f,N,a,b)
> local TV,TR,k,av,ap,T,C;
> TV:=NULL: TR:=NULL:
> for k from 0 to N-1 do
> av:=a+k*(b-a)/N:
> ap:=a+(k+1)*(b-a)/N;
> T:=[[av,0],[av,f(av)],[ap,f(ap)],[ap,0]]:
> TV:=TV,plot(T,filled=true,color=wheat):
> TR:=TR,plot(T,color=black,thickness=2):
> od:
```
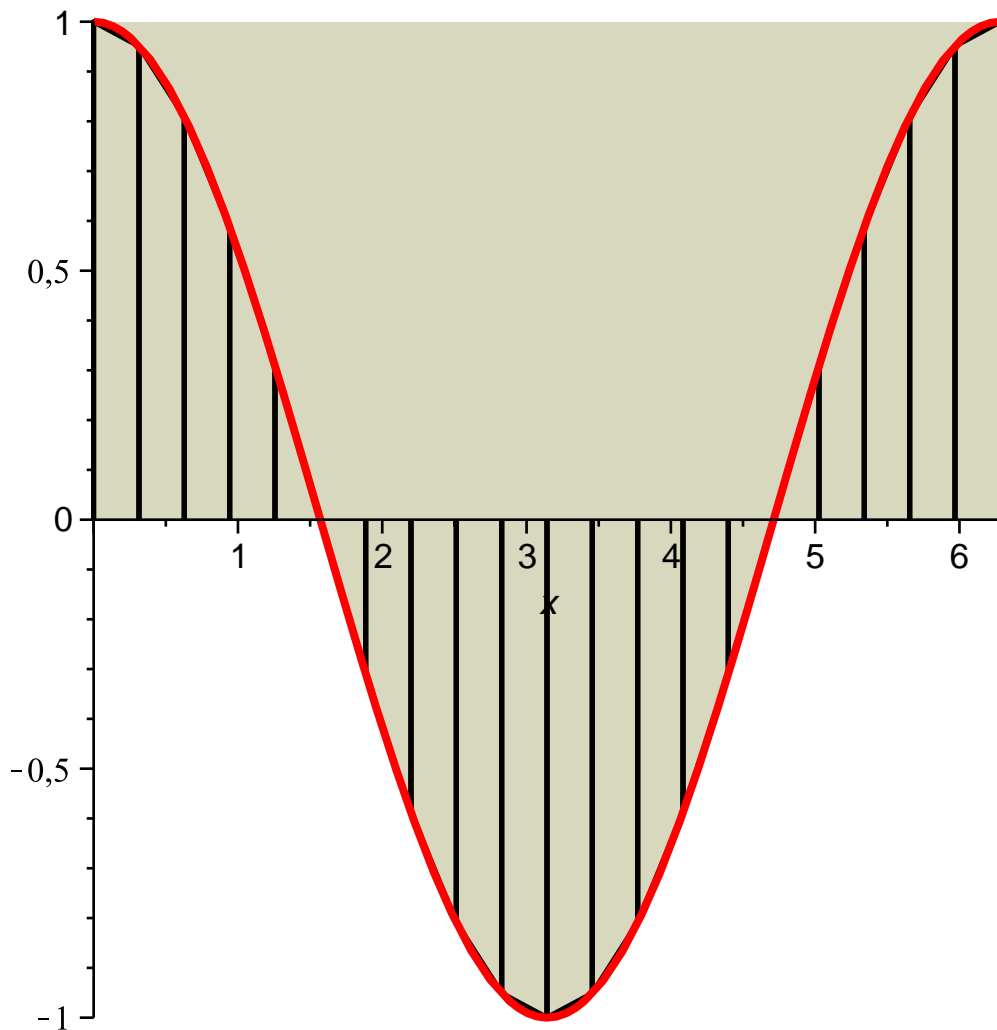
```
> C:=plot(f(x),x=a..b,thickness=3):
> plots[display](TV,TR,C);
> end:

> Trap(x->cos(x),10,0,Pi);
```



```
> Trapanim:=proc(f,N,a,b)
> local nbtrap,i,S,n;
> nbtrap:=[seq(i,i=1..N)]:
> S:=seq(Trap(f,n,a,b),n=nbtrap):
> plots[display](S,insequence=true);
> end:

> Trapanim(x->cos(x),20,0,2*Pi);
```

```
> Sims:=proc(f,a,b,dx)
  local aa,S;
  aa:=a;
  S:=0;
  while(evalf(aa)<evalf(b)) do
    S:=S+(f(aa)+4*f(aa+0.5*dx)+f(aa+dx))*dx/6.;
    aa:=aa+dx;
  od;
  S;
  end:
```

```
> Sims(x->sin(x),0,Pi,0.0001);
```

1.99999999997301512860656076131829841819292467301705738816084111709735552384681\   **(2)**
    09544655922413037065226

```
> Riem:=proc(f,a,b)
> local k;
```

```
> Limit(Sum(((b-a)/n)*f(a+k*(b-a)/n),k=1..n),n=infinity)=limit
  (sum(((b-a)/n)*f(a+k*(b-a)/n),k=1..n),n=infinity);
> end:
```

```
> Riem(x->sin(x),0,Pi);
```

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{\pi\sin\left(\frac{k\,\pi}{n}\right)}{n}\right)=2 \tag{3}$$

```
> airetrap:=proc(f,N,a,b)
> local k:
> (b-a)/N*((f(a)+f(b))/2+Sum(f(a+k*(b-a)/N),k=1..N-1))=evalf((b-
  a)/N*((f(a)+f(b))/2+Sum(f(a+k*(b-a)/N),k=1..N-1)));
> end:
```

```
> evalf(Pi);
```
$$3.141592654 \tag{4}$$

```
> 4*airerect(x->sqrt(1-x^2),10000,0,1);
```

$$4\left(\sum_{k=1}^{10000}\frac{1}{100000000}\sqrt{100000000-k^2}\right)=3.141391481 \tag{5}$$

```
> 4*airetrap(x->sqrt(1-x^2),10000,0,1);
```

$$\frac{1}{5000}+\frac{1}{2500}\sum_{k=1}^{9999}\frac{1}{10000}\sqrt{100000000-k^2}=3.141591478 \tag{6}$$

```
> mini:=proc(u,p)
> local k;
> k:=0;
> while (evalf((u^(2*k+3))/(2*k+3)))>(10^(-p)) do
> k:=k+1;
> od:
> end:
```

```
> greg:=proc(n,a,p)
> local S,k;
> S:=0;
> for k from 0 to n+1/2 do
> S:=S+evalf(((-1)^k*a^(2*k+1))/(2*k+1),p+1);
> od;
> S;
> end:
```

```
> mini(1/5,100);
```
$$69 \tag{7}$$

```
> Digits:=100:
> 4*(4*greg(69,1/5,100)-greg(69,1/239,100));
```

$$\tag{8}$$

3.14159265358979323846264338327950288419716939937510582097494459230781640628 62\
08998628034825342117064 **(8)**

> `4*(4*greg(69,1/5,100)-greg(69,1/239,100))-evalf(Pi,100);`

$$-4. \, 10^{-99}$$ **(9)**