

Exercice I

$$1. z_1 z_2 = 3\sqrt{2} + \sqrt{6} + i(3\sqrt{2} - \sqrt{6})$$

$$2. z_1 = 2\sqrt{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = 2\sqrt{2} e^{-i\frac{\pi}{6}}$$

$$z_1 z_2 = 2\sqrt{2}\sqrt{6}e^{i\frac{\pi}{4}-i\frac{\pi}{6}} = 4\sqrt{3}e^{i\frac{\pi}{12}}$$

$$\frac{z_1}{z_2} = \frac{\sqrt{2}(\sqrt{3}-i) \times \sqrt{3}(1-i)}{3+3} = \frac{\sqrt{6}}{6} (\sqrt{3}-1+i(-1-\sqrt{3}))$$

$$z_2 = \sqrt{6} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) = \sqrt{6}e^{i\frac{\pi}{4}}$$

$$\frac{z_1}{z_2} = \frac{2\sqrt{2}}{\sqrt{6}} e^{-i\frac{\pi}{4}-i\frac{\pi}{6}} = \frac{2\sqrt{3}}{3} e^{-i\frac{5\pi}{12}}$$

Exercice II

On utilise les formules d'Euler :

```
trig2exp(cos(x)); trig2exp(sin(x))
```

$$\frac{(e^{ix} + \frac{1}{e^{ix}})}{2}, \frac{(e^{ix} - \frac{1}{e^{ix}})}{2 * i}$$

et les formules de développement :

```
expand((a+b)^3); expand((a-b)^2)
```

$$3a^2b + 3ab^2 + b^3 + a^3, -(2ab) + (-b)^2 + a^2$$

et on obtient

```
tcollect((cos(x))^3)
```

$$\frac{3}{4} \cdot \cos(x) + \frac{1}{4} \cdot \cos(3x)$$

```
tcollect((sin(2*x))^2)
```

$$\frac{1}{2} + \frac{1}{-2} \cdot \cos(4x)$$

Exercice III

On résout l'équation :

```
S:=csolve(z^2-2*sqrt(2)*z+4=0, z)
```

$$[\sqrt{2} \cdot i + \sqrt{2}, -(\sqrt{2} \cdot i) + \sqrt{2}]$$

On cherche module et argument de la première solution :

```
abs(S[0]); arg(S[0])
```

$$2, \frac{\pi}{4}$$

puis de la deuxième :

```
simplifier(abs(S[1])); simplifier(arg(S[1]))
```

$$2, \frac{(-\pi)}{4}$$

Exercice IV

```
z1 := 2*exp(i*pi/4)
```

$$2\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)$$

```
z2 := 3*exp(-i*pi/6)
```

$$3\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$$

```
z1+z2
```

$$2\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right) + 3\left(\frac{\sqrt{3}}{2} + \frac{-i}{2}\right)$$

```
approx(abs(z1+z2))
```

$$4.013207$$

```
approx(arg(z1+z2))
```

```
Psr2, Mod5.74456, Heu1.6, Min1.6GCDdim1, n=10maxpqdeg04(Vector[4])//UsingHeugcd//mrrefline0
```

$$-0.021378$$